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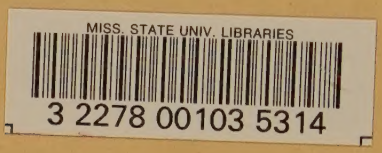

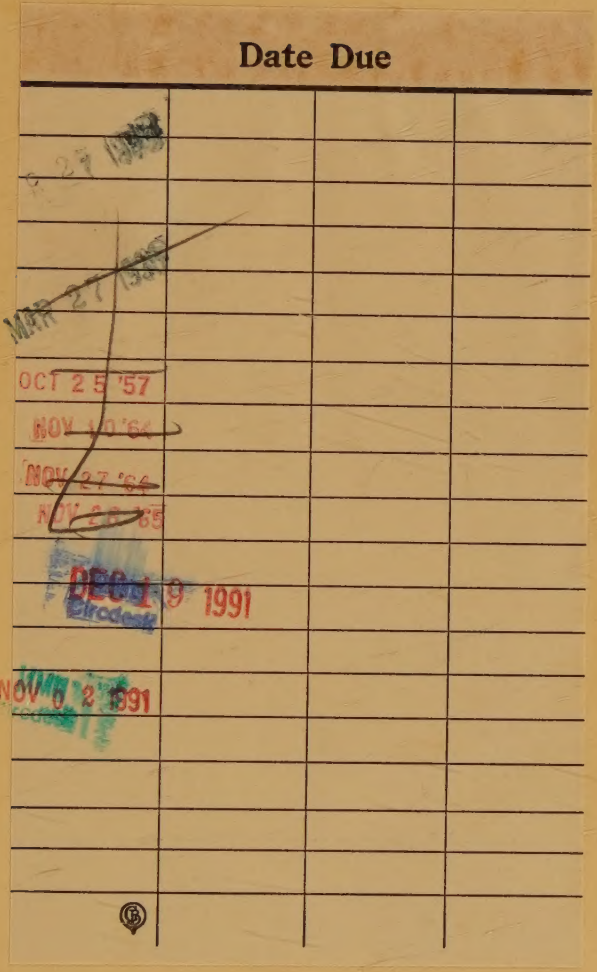
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ELECTRIC TRANSIENTS

ELECTRIC TRANSIENTS

BY

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PREFACE TO SECOND EDITION

The second edition is essentially a new book. All the oscillograms illustrating the text are new and of superior quality. Every chapter has been revised and one chapter has been added. The book is an outline of an introductory course in electric transients given by the author for the past 16 years to electrical engineering seniors in the University of Washington. The purpose of the book is to aid students in gaining clear physical concepts of the fundamental principles underlying transient electric phenomena and their application to quantitative problems.

It is generally recognized that the main purpose of engineering students while in college should be to gain insight into, and understanding of, the *fundamentals of engineering*, that is, the basic physical laws, by and under which engineers work, live and have their being; and to acquire a fair working ability to apply these laws to quantitative practical problems. There can be no question of the far-reaching importance of transient electric phenomena in all branches of electrical engineering. Electromagnetic-dielectric stresses resulting from sudden changes in power circuits, as the starting of generators or motors, breaking of field circuits, spark-overs, lightning discharges and other disturbances, are of great importance both in the design of electrical appliances and in the operation of electric power systems. In fact, it is fully as important that graduates in electrical engineering should have a fair working knowledge of the fundamental principles of transient electric phenomena as of the more widely known laws of direct and alternating currents that apply under constant or permanent conditions.

The course as outlined is professedly of an elementary nature, with emphasis on the energy content and the

physical characteristics of electric transients. The text is illustrated and supplemented by a large number of oscillograms of electric transients that occur in the various types of electric machines and circuits in common use in electrical engineering laboratories. The problems are largely based on quantitative data obtained from laboratory experiment under circuit conditions definitely specified, so that students may reproduce the oscillograms, check the results and gain that perspective of quantitative relations so essential to all engineers. Adjusting and operating an oscillograph and producing sharply defined and well-proportioned oscillograms are effective methods for acquiring clear concepts of quantitative values, both relative and absolute, of the factors involved in transient electric phenomena. It is advisable to require the student to devote at least two-thirds of the time allotted to a course in electric transients to the taking of oscillograms. Instructions for operating the oscillograph are given in Chap. X. In order to produce good oscillograms, just as painstaking care is required in handling the films and in developing and printing the photographic record as in adjusting and operating the oscillograph.

The illustrations form an outstanding feature of the book. The oscillograms were all taken in the electrical engineering laboratories of the University of Washington by my colleague, Asst. Professor George S. Smith, and by students in the course.

The author desires to express his appreciation of the valuable assistance rendered by Asst. Prof. Smith, and for helpful suggestions from Prof. Stewart W. Anderson, Mr. J. R. Tolmie, Mr. A Kalin and Prof. E. A. Loew.

C. Edward Magnusson.

University of Washington,
Seattle, Wash.,
May, 1926.

PREFACE TO FIRST EDITION

Transient electric phenomena generally increase in commercial importance with the size and complexity of electric systems, and a knowledge of the fundamental principles of electric transients and their application to the solution of quantitative problems is as essential to the successful operation of large power and communication systems as a mastery of the basic laws of direct and alternating currents.

This work is an outline of an introductory lecture and laboratory course given during the past twelve years to electrical engineering students in the University of Washington. The purpose of the book is to aid the student in gaining clear concepts of the fundamental principles of electric transient phenomena and their application to quantitative problems. The course as outlined is professedly of an elementary character with emphasis placed on the physical properties of electric trnasients. The text is illustrated and supplemented by a large number of oscillograms of transients that occur in the various types of machines and electric circuits in common use in electrical engineering laboratories. The problems are based on quantitative data obtained from laboratory experiments under circuit conditions that may easily be reproduced by the student.

Quantitative laboratory work is essential in order to readily gain insight into the physical nature of transient electric phenomena. It is advisable to require the student to devote at least two-thirds of the time allotted to a course in electric transients to the taking of oscillograms. Adjusting an oscillograph so as to obtain sharply defined, well proportioned oscillograms of electric transients is an effec-

tive method for acquiring due appreciation of quantitative values, both absolute and relative, of the factors involved. The quality of the photographic record depends as much on painstaking care in handling the films and in developing and printing the oscillograms as on skilful operation of the oscillograph. Many pitfalls in the photographic part of the work may be avoided by carefully following the directions given in the Appendix.

No attempt is made to give references to original investigations or to papers and books dealing with the various phases of electric transient phenomena, as the principles discussed are well established and the material is arranged in text book form. A distinctive feature of the book lies in the illustrations. All of the oscillograms were taken by A. Kalin and J. R. Tolmie or by students in the course under their direction in the electrical engineering laboratories of the University of Washington.

C. Edward Magnusson.

Seattle, Wash.,
March, 1922.

CONTENTS

PAGE

PREFACE TO SECOND EDITION v

PREFACE TO FIRST EDITION vii

CHAPTER I.—INTRODUCTION 1
Magnetic circuit—Dielectric circuit—Electric circuit.

CHAPTER II.—INSTRUMENTS 10
Spark gaps—Single, three and six element oscillographs—Braun tube oscillographs—Cathode ray oscillographs—The klylonograph—Oscillator generator—Lichtenberg figures—Timing wave—Oscillograms—Problems and experiments.

CHAPTER III.—SINGLE-ENERGY TRANSIENTS. DIRECT CURRENTS. 46
Single-energy circuits—Fundamental relations—The damping factor—The time constant—Dissipation or attenuation constants—The exponential curve—Initial transient values—Current, voltage and magnetic flux transients—Problems and experiments.

CHAPTER IV.—SINGLE-ENERGY TRANSIENTS. ALTERNATING CURRENTS 66
Single-phase, single-energy load circuit transients—Three-phase, single-energy load circuit transients—Starting transient of a polyphase rotating magnetic field—Polyphase short circuits. Alternator armature and field transients—Single-phase short circuits. Alternator armature and field transients—Single-phase short circuits on polyphase alternators—Problems and experiments.

CHAPTER V.—DOUBLE-ENERGY TRANSIENTS 106
Double-energy circuits—Surge or natural impedance and admittance—Frequency of oscillations in double-energy circuits—Dissipation constant and damping factor in simple double-energy circuits—Equations for current and voltage transients—Problems and experiments.

CHAPTER VI.—ELECTRIC-LINE OSCILLATIONS. SURGES AND TRAVELING WAVES 131
Artificial transmission lines—Time, space and phase angles—Natural period of oscillation—Length of line—Velocity unit of length—Surge impedance—Voltage and current oscillations and

PAGE

power surges—General transmission-line equations—Traveling waves—Compound circuits—Problems and experiments.

CHAPTER VII.—VARIABLE CIRCUIT CONSTANTS 166
Variable resistance—Variable inductance—Variable conductance—Variable condensance—Problems and experiments.

CHAPTER VIII.—RESONANCE 184
Voltage resonance—Reactance curves—Current resonance—Coupled circuits—Direct coupling—Inductive coupling—Condensive coupling—Coupling coefficient—Multiplex resonance—Resonance growth and decay—Problems and experiments.

CHAPTER IX.—OSCILLOGRAMS. KLYDONOGRAMS 205
A series of oscillograms illustrating various forms of electric transients in electric power circuits and machinery—Cathode-ray oscillograms—Klydonograms—Lichtenberg figures.

CHAPTER X.—LABORATORY INSTRUCTIONS 225
Operating oscillographs. Developing and printing oscillograms.

INDEX 237

ELECTRIC TRANSIENTS

CHAPTER I

INTRODUCTION

The laws for direct currents, as usually expressed, state the relations of the several factors involved under continuous or permanent conditions, and cannot be correctly applied while the current or voltage is increasing or decreasing. Similarly, alternating currents are expressed as continuous phenomena by means of effective values and complex quantities, on the basis that the successive cycles are of the same magnitude and wave shape. Observations and test data for both the direct-current and alternating-current systems are ordinarily taken only during steady or permanent conditions. The equations derived, and the data obtained from tests, apply only to permanent or constant conditions and can not be correctly applied during transition periods when the conditions vary. Transient electric phenomena, as the term implies, are usually of short duration and relate to what occurs in an electric circuit between periods of stable conditions. This definition is, however, not rigidly adhered to in electrical discussions. Frequently other disturbances that cause damage and trouble in the operation of electric systems, such as unstable electric equilibrium, permanent instability, resonance and cumulative oscillations, are included with the true transients under the caption of transient electric phenomena.

It is important that the student should realize that electric transients are of very frequent occurrence in all commercial electric systems. Any change, such as the starting or stopping of a motor, the turning on of a lamp, or any change in the operating conditions, necessitates a

readjustment of the energy content in the whole system and produces electric transients just as truly as a stroke of lightning or a short circuit. In the operation of street car systems the changes in load, and hence the transients on the system, are so frequent that they often overlap and occupy by far the greater part of the time; hence, for street railway systems, it might appear simpler to define the permanent or steady conditions as short periods occurring between successive series of overlapping transients.

Electrical engineering deals with the transmission and transformation of electric energy. During permanent conditions the flow of energy is uniform and continuous; any change in the power indicates a transient condition. Changes in the current and voltage factors imply a corresponding change in the energy content of the electric field, since a magnetic field surrounds all electric currents, and an increase or a decrease in the current necessitates a corresponding change in the stored magnetic energy. Similarly, any change in voltage between conductors must be accompanied by a corresponding readjustment in the energy stored in the dielectric field of the system.

Magnetic Circuit.—In the study of transient phenomena, as well as of all phases of the electric field, Faraday's concept of magnetic and dielectric lines of force is of fundamental importance. All magnetic lines are continuous and closed on themselves. Ohm's law applies to the magnetic circuits in the same way as to the electric circuit. The magnetic flux produced is equal to the magnetomotive force divided by the reluctance.

$$\text{Magnetic flux} = \frac{\text{magnetomotive force}}{\text{reluctance}};$$

$$\Phi = \frac{\mathfrak{F}}{\mathfrak{R}} \text{ or } \mathfrak{F} = \mathfrak{R}\Phi \qquad (1)$$

The magnetic field is produced by, and is proportional to, the electric current.

$$\Phi = Li \qquad (2)$$

The proportionality factor L is called the inductance of the circuit.

The reluctance varies directly as the length and inversely as the cross-section of the magnetic circuit. The specific reluctance per cm.^{3} is the reciprocal of the permeability μ. If the magnetomotive force is expressed in ampere-turns, the resultant field intensity is given by the equation

$$H = 4\pi nI\ 10^{-1} \text{ per cm.} \qquad (3)$$

This magnetizing force produces a magnetic flux density of B lines per cm.^{2} in materials having μ permeability.

$$B = \mu H \text{ lines per cm.}^2 \qquad (4)$$

The permeability is the reciprocal of the specific reluctance in the magnetic circuits and corresponds to the specific

FIG. 1.—Magnetic field of single conductor.

FIG. 2.—Magnetic field of circuit.

conductivity of the conductor in the electric circuits. In empty space $\mu = 1$ and for all non-magnetic materials it is very nearly equal to unity. For magnetic materials the permeability is greatly increased and may reach several thousand for soft iron and steel. The factor 4π comes from the definition of a unit magnetic pole as having one line per cm.^{2} on the surface of a sphere of unit radius. The 10^{-1} factor results from the definition of the ampere.

In building up a magnetic field, lines of force cut the conductor and thus produce a counter e.m.f., or inductance

voltage ${}_{L}e$, which is equal to the time rate of change of the interlinked magnetic flux.

$$ {}_{L}e = -\frac{d\Phi}{dt} = -L\frac{di}{dt} \tag{5} $$

Necessarily, an equal opposite voltage must be impressed to force the current through the electric circuit. The product of the voltage and the current represents the power required to generate the field. Hence, the energy stored in a magnetic field by a current I, in a circuit having an inductance L, is given by equations (6) and (7).

$$ \int_{o}^{W} dw = -\int {}_{L}eidt = -L\int_{o}^{i} idi \tag{6} $$

$$ W = \frac{Li^2}{2} \tag{7} $$

The energy is stored magnetically in the electric field surrounding the conductor and is proportional to the square of the current. When the current decreases the energy is returned to the circuit, for if i and therefore ϕ decrease, di/dt and hence ${}_{L}e$ are negative, which means that the energy is returned to the electric circuit.

The practical unit of inductance L is the henry. In any consistent system of units a circuit possesses one unit of inductance if a unit rate of change of current in the circuit generates or consumes one unit of voltage. If the current changes *at the rate of one ampere per second*, and the voltage generated or consumed is *one volt*, then the inductance is *one henry*.

Dielectric Circuit.—For the dielectric field similar relations exist. All dielectric lines of force are continuous and end on conductors. Ohm's law may be applied to the dielectric circuit in the same manner as to the magnetic and electric circuits.

$$ \text{Dielectric flux} = \frac{\text{voltage}}{\text{elastance}}; $$

$$ \Psi = \frac{e}{S} = Ce \tag{8} $$

The dielectric flux is directly proportional to the voltage between the conductors and inversely proportional to the elastance of the dielectric circuit. The elastance S is the reciprocal of the condensance C and varies directly as the length x and inversely as the cross-section A of the dielectric circuit. It corresponds to resistance of the electric circuit and to reluctance of the magnetic circuit.

FIG. 3.—Dielectric field of single conductor.

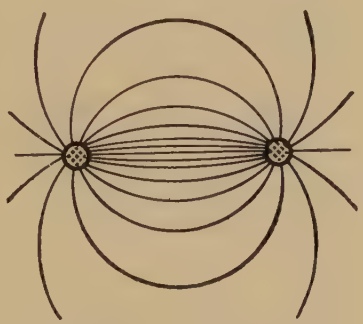

FIG. 4.—Dielectric field of circuit.

$$S = \frac{4\pi x}{\kappa A};\ C = \frac{\kappa A}{4\pi x} \text{ in c.g.s. electrostatic units} \tag{9}$$

$$S = \frac{4\pi v^2 x}{\kappa A};\ C = \frac{\kappa A}{4\pi v^2 x} \text{ in c.g.s. electromagnetic units} \tag{10}$$

$$S = \frac{4\pi v^2 x}{\kappa A 10^9} = 113.1\,\frac{10^{11} x}{\kappa A} \text{ darafs} \tag{11}$$

$$C = \frac{\kappa A 10^9}{4\pi v^2 x} = 88.42\,\frac{\kappa A}{x} 10^{-15} \text{ farads} \tag{12}$$

$$C = 88.42\,\frac{\kappa A}{x} 10^{-9} \text{ mf.} \tag{13}$$

The permittivity κ is unity for empty space and very nearly equal to unity for air and many other materials. In Table I is given the permittivity constants for the more common dielectrics used in electric apparatus. The constant $v = 3 \cdot 10^{10}$ cm. per sec., the velocity of the propagation of an electric field in space (equivalent to the velocity of light), is the ratio of the units used in the electromagnetic

and electrostatic systems. The factor 4π comes from the definition of a unit line of dielectric force.

TABLE I

Material	Permittivity	Material	Permittivity
Air and other gases	1.0	Olive oil	3.0 to 3.2
Alcohol, amyl	15.0	Paper with turpentine	2.4
Alcohol, ethyl	24.3 to 27.4	Paper or jute impregnated	4.3
Alcohol, methyl	32.7	Paraffin	2.3
Asphalt	4.1	Paraffin oil	1.9
Bakelite	6.6 to 16.0	Petroleum	2.0
Benzine	1.9	Porcelain	5.3
Benzol	2.2 to 2.4	Rubber	2.4
Condensite	6.6 to 16.0	Rubber vulcanized	2.5 to 3.5
Glass (easily fusible)	2.0 to 5.0	Shellac	2.7 to 4.1
Glass (difficult to fuse)	5.0 to 10.0	Silk	1.6
Gutta-percha	3.0 to 5.0	Sulphur	4.0
Ice	3.0	Turpentine	2.2
Marble	6.0	Varnish	2.0 to 4.1
Mica	5.0 to 7.0		
Micarta	4.1		

The charging current ${}_ci$ storing energy in the dielectric circuit is equal to the time rate of change in the dielectric flux

$$ {}_ci = -\frac{d\Psi}{dt} = -C\frac{de}{dt} \tag{14}$$

Hence the energy stored in the dielectric field by a voltage E in a circuit having a condensance C is given by equations (15) and (16):

$$\int_o^W dw = -\int {}_cie dt = -C\int_o^e ede \tag{15}$$

$$W = \frac{Ce^2}{2} \tag{16}$$

The energy stored dielectrically in the electric field surrounding a conductor is proportional to the square of the voltage. When the voltage decreases the energy is returned to the electric circuit, for if e and therefore ψ decrease, then de/dt and hence ${}_ci$ are negative, which

means that the energy is returned to the electric circuit. The unit of condensance (capacitance) C is the farad. In any consistent system of units a circuit possesses one unit of condensance if a unit rate of change of voltage produces (or consumes) one unit of current. If the voltage changes *at the rate of one volt per second* and the current produced (or consumed) is *one ampere*, the condensance

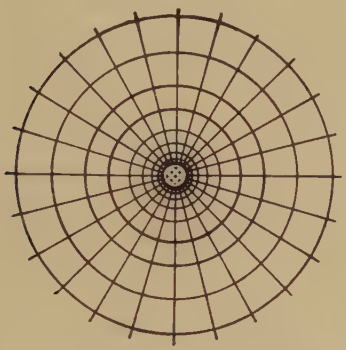

FIG. 5.—Electric field of conductor.

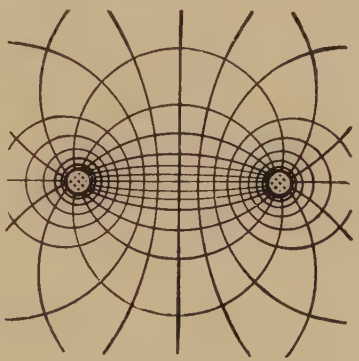

FIG. 6.—Electric field of circuit.

of the circuit is *one farad*. The farad is too large a unit for practical purposes and hence in commercial problems the condensance is usually measured in microfarads.

$$1 \text{ farad} = 10^6 \text{ mf.} \tag{17}$$

Electric Circuit.—The electric circuit relates specifically to the conductor carrying the electric current, although the term is frequently made to include the dielectric and magnetic fields, since the electric, dielectric and magnetic circuits are interlinked. Under steady or permanent conditions in a direct-current system the electric circuit transmits the energy without causing any change in the energy stored magnetically and dielectrically in the space surrounding the electric circuit. In starting the system a transient condition exists until the magnetic and dielectric fields have been supplied with the required amount of energy as determined by the magnitude of the current and voltage and the circuit constants.

If the electric circuit be considered as something separate and apart from the surrounding magnetic and dielectric fields, no storage of energy would be involved and hence no transients could exist, since all the changes would be instantaneous. But the electric circuit is interlinked with the dielectric and magnetic circuits. Changes in the current and voltage in the electric circuit are accompanied by changes in the energy stored in the dielectric and magnetic fields, thus necessitating a readjustment of the energy content in the whole electric system. The transfer of energy requires time and thus the transient period is of definite, although often of extremely short, duration.

The close analogy existing between electric, dielectric and magnetic circuits may be shown to advantage by arranging the corresponding quantities in tabular form as in Table II.

For convenience in solving problems the energy equations are expressed in the units used in commercial work:

Energy in a magnetic field:

$$= W(\text{joules}) = \frac{L(\text{henrys})\ i^2\ (\text{amp.})}{2} \qquad (18)$$

Energy in a dielectric field:

$$= W(\text{joules}) = \frac{C(\text{mf.})\ e^2\ (\text{volts})}{2 \times 10^6} \qquad (19)$$

Energy in a moving body:

$$= W(\text{ergs}) = \frac{M(\text{g.})\ v^2\ (\text{c.m. per sec.})}{2} \qquad (20)$$

Energy in a moving body:

$$= W(\text{joules}) = \frac{M(\text{kg.})\ v^2\ (\text{meters per sec.})}{2}$$

Energy in a moving body:

$$= W(\text{ft.-lb.}) = \frac{M(\text{lb.})\ v^2\ (\text{ft. per sec.})}{2 \times 32.2}$$

1 joule = 1 watt-sec. = 10^7 ergs = 0.7376 ft.-lb.
= 0.2389 g.-cal. = 0.102 kg.-m. = 0.0009480 B.t.u. (21)

1 ft.-lb. = 1.356 joules = 0.3239 g. = 0.1383 kg.-m.
= 0.001285 B.t.u. = 0.0003766 watt-hr. (22)

1 B.t.u. = 1,055 joules = 778.1 ft.-lb. = 252 g.-cal.
= 0.2930 watt-hr. (23)

TABLE II

Electric circuit	Dielectric circuit	Magnetic circuit
Electric current: $i = Ge = \frac{e}{R}$ amperes.	Dielectric flux (dielectric current): $\Psi = Ce = \frac{e}{S}$ lines of dielectric force.	Magnetic flux (magnetic current): $\phi = Li\ 10^8$ lines of magnetic force, maxwells. One line = one maxwell.
Electromotive force, voltage: e = volts.	Electromotive force: e = volts.	Magnetomotive force. $\mathfrak{F} = 4\pi ni$ amp.-turns. gilberts.
Conductance: $G = \frac{i}{e}$ mhos.	Condensance, capacitance, permittance or capacity $C = \frac{\Psi}{e}$ farads.	Inductance: $L = \frac{n\phi}{\mathfrak{F}} 10^{-8} = \frac{\phi}{i} 10^{-8}$ henrys.
Resistance: $R = \frac{e}{i}$ ohms.	Elastance: $S = \frac{1}{C} = \frac{e}{\Psi}$ darafs.	Reluctance: $R = \frac{\mathfrak{F}}{\phi}$ oersteds.
Electric power: $p = Ri^2 = Ge^2 = ie$ watts.	Dielectric energy: $w = \frac{Ce^2}{2} = \frac{\Psi e}{8\pi}$ joules.	Magnetic energy: $w = \frac{Li^2}{2} = \frac{\phi i}{2} 10^{-8}$ joules.
Electric-current density: $I = \frac{i}{A} = \gamma G'$ amp. per cm.^{2}	Dielectric-flux density: $D = \frac{\Psi}{A} = \kappa K$ lines per cm.^{2}	Magnetic-flux density: $B = \frac{\phi}{A} = \mu H$ lines per cm.^{2}
Electric gradient: $G' = \frac{e}{l}$ volts per cm.	Dielectric gradient $G' = \frac{e}{l}$ volts per cm.	Magnetic gradient: $f = \frac{\mathfrak{F}}{l}$ amp.-turns per cm.
Conductivity: $\gamma = \frac{I}{G'}$ mho per cm.^{3}	Condensivity, permittivity or specific capacity: $k = \frac{D}{K}$	Permeability: $\mu = \frac{B}{H}$
Resistivity: $\rho = \frac{1}{\gamma} = \frac{G'}{I}$ ohm per cm.^{3}	Elastivity: $\frac{1}{k} = \frac{K}{D}$	Reluctivity: $v = \frac{f}{B}$
Specific electric power: $p = \rho I^2 = \gamma G'^2 = G'I$ watts per cm.^{3}	Specific dielectric energy: $w = \frac{kG'^2}{4\pi v^2} = \frac{G'D}{2}$ joules per cm.^{3}	Specific magnetic energy: $w = \frac{0.4\pi\mu f^2}{2} = \frac{fB}{2} 10^{-8}$ joules per cm.^{3}
	Condensance, permittance, capacitance current: $ci = \frac{d\Psi}{dt} = C\frac{de}{dt}$ amp.	Inductance voltage: $e = \frac{d\phi}{dt} 10^{-8} = L\frac{di}{dt}$ volts.
	Dielectric-field intensity: $K = \frac{G'}{4\pi v^2}$ lines of dielectric force per cm.^{2}	Magnetic-field intensity $H = 4\pi f 10^{-1}$ lines of magnetic force per cm.^{2}

CHAPTER II

INSTRUMENTS

When taking experimental data on transient electric phenomena ordinary voltmeters, ammeters, wattmeters and frequency indicators become accessory apparatus, while quantitative measurements of primary importance are obtained by means of oscillographs, the klydonograph and by various forms of spark gaps.

Spark Gaps.—With the development of several types of oscillographs, klydonograph and other forms of measuring

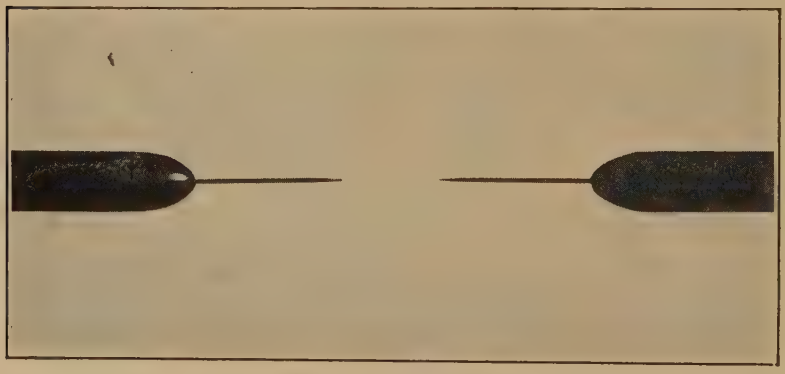

Fig. 7.—Needle gap.

devices, spark gaps have to a considerable extent been superseded as instruments for taking quantitative data on electric transients. Three forms, the needle, horn and sphere spark gaps, are used both as protective appliances and for measuring high-tension voltage, as, for example, in making flash-over tests on transmission-line insulators or transformer bushings.

The needle gap, Fig. 7, consists of a pair of sewing needles (No. 00 double long are standard, A. I. E. E. rule No. 2365) held in position by supporting shanks. The size of the

needle, the shape of supports, distance from ground, atmospheric conditions and, to a marked degree, the form of the voltage impulse to be measured affect the spark-over distance, or setting of the gap, between the needle points. Even with careful calibration the needle-gap readings taken on transient electric impulses are of questionable value as quantitative data.

The horn gap, Fig. 8, is in general use on power systems as a protective device in combination with some form of lightning arrester. Two wires or bars, usually of copper,

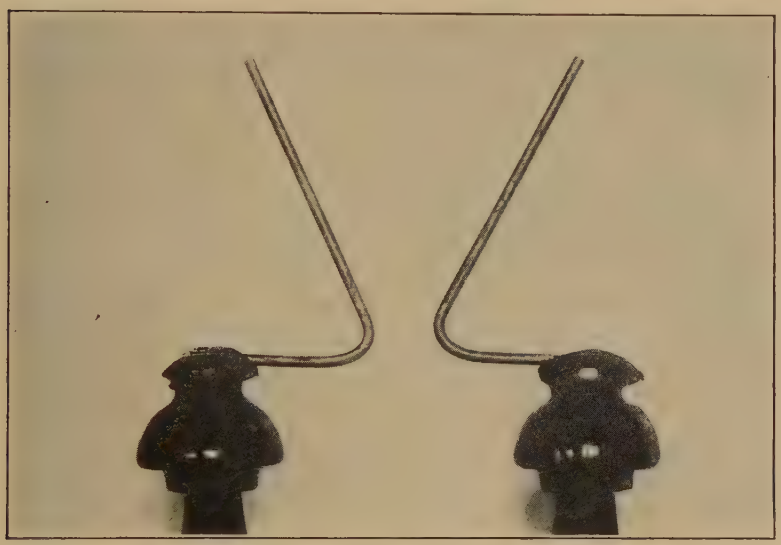

FIG. 8.—Horn gap.

are bent into shapes somewhat resembling rams' horns. When arc-over occurs the reactive magnetic field around the horns and in the arc itself forces the arc upwards and thereby increases its length until the arc ruptures. The heat generated by the arc augments the force produced by the magnetic field and hastens the extinction of the arc.

The sphere gap is similar in construction to the horn gap except that two spheres or hemispheres are used in place of the curved wires or horns, as illustrated in Fig. 9.

When constructed and used as specified in accord with A. I. E. E. Standards, rule No. 2366, the accuracy obtainable should be within 2 per cent. The difference as well as the similarity of the electrical characteristics of the sphere gap, horn gap and needle gap can best be made clear by briefly reviewing the basic principles of electric conduction through gases.

"When[1] a spark gap in air is excited from a source of potential, the air in the vicinity of the gap electrodes becomes ionized, to a degree depending on the intensity of the

Fig. 9.—Sphere gap.

electric field. The ionization of the air, which means the tearing apart of the atoms or molecules into positive and negative parts, begins at a definite gradient, or for a given gap at a definite voltage. This tearing apart of the air consumes energy, and since energy involves a time factor, time is required to ionize the air. As soon as the ionization has produced a saturated path of conducting ions from one electrode to the other, a discharge takes place along this conducting path, that is, the gap arcs over. The interval of time elapsing from the instant when the voltage reached the breakdown value until the moment of spark-over is called the *time lag*.

[1] Bulletin—G. E. Co.

If a uniform field exists across the air gap, the whole space will be subjected to the same gradient, and hence ionization will occur uniformly throughout the whole length of the gap, as soon as the field intensity has reached the critical value. The *time lag* of such a gap will be a minimum, and spark-over will occur as soon as the necessary disruptive energy can be delivered. It is obvious that a form of gap which does not establish a uniform field will not be uniformly ionized from the instant the critical voltage is reached. At first, a saturated field will be produced in the region of greatest intensity, as, for instance, at the points of a needle gap, and discharge (corona) will take place from the electrodes across this saturated region to the non-ionized space beyond. The corona is a conductor and its formation virtually enlarges the electrodes until, finally, with the increase of voltage, a path of good conductivity is established across the entire gap. Spark-over then follows. The time required to break down such a gap progressively is longer than the simultaneous breakdown the whole distance across a uniform field gap, *i.e.*, its *time lag* is greater. Although this time interval is actually very small, yet, in comparison with the time required in the uniform field gap, it is relatively large. Therefore the time lag of the uniform field gap is small, while the time lag of the non-uniform field gap is comparatively large.

The voltages which can be applied to a gap vary greatly in their rate of change. For instance, a 60-cycle sine wave changes very slowly in comparison with a lightning impulse. In terms of micro-seconds (millionths of a second), it takes nearly 4,000 such time units for a 60-cycle wave to change from zero to its maximum value. A 1,000,000-cycle impulse would require only ¼ micro-sec. to reach its peak. If the voltage across a gap has risen slowly during the ionization process so that the breakdown value finally reached has prevailed for a relatively long time (as, for instance, a 60-cycle sine wave), the spark-over will have time to occur before the voltage increases

appreciably above the critical value. The moment of spark-over will follow the instant when breakdown voltage is reached by some definite time interval, *i.e.*, by a certain time lag for this kind of a voltage wave. On the other hand, if the voltage has risen rapidly, and continues to rise rapidly after the 60-cycle breakdown value is passed, the time required to saturate the gap will be different and less than before, although during this interval the voltage greatly exceeds the 60-cycle breakdown value. This will be the result when an impulse of steep wave front and of a maximum voltage much higher than the 60-cycle breakdown value is applied. At the moment when the 60-cycle breakdown value is reached the impulse wave is increasing so rapidly that it rises to a much higher value before the necessary time elapses to establish a flash-over.

It is therefore apparent that *time lag* depends on both the character of the gap and the nature of the voltage wave. Since in a uniform field gap the ionization occurs simultaneously throughout the field, it is obvious that spark-over will occur in such a field practically as soon as the critical voltage is reached, regardless of whether this voltage is relatively constant or rapidly increasing. In other words, since the time lag of such a gap is so very small, even with respect to a high-frequency wave, the breakdown occurs at essentially the same value of the voltage, regardless of its rate of change. Such a gap is approximated by two smooth spheres of proper dimensions placed close together. A sphere gap therefore sparks over at approximately the same voltage value, independent of frequency. In the case of the non-uniform field gap, however, the time lag is so much greater than the uniform field gap that rapidly rising voltages have time to reach greater values than the 60-cycle spark-over value before ionization is completed and the gap discharges. This higher value of spark-over voltage obtained with a high-frequency impulse is commonly called the *impulse spark-over* in distinction to the 60-*cycle spark-over* value. The ratio of the *impulse spark-*

over to the 60-*cycle spark-over* is termed the *impulse ratio.* This impulse ratio is dependent upon the time lag of the gap, and consequently upon the kind of gap and form of voltage wave.

To restate these principles: (1) A uniform field gap has small and negligible time lag, and is called *fast.* It discharges low-frequency (or continuously applied) voltages and high-frequency (or steep wave front) impulses at the same value; *i.e.*, its 60-cycle spark-over and impulse spark-over are the same. The *impulse ratio* is therefore unity. A sphere gap approximates this type. (2) A non-uniform field gap has large and influential time lag, and is called *slow.* Its impulse spark-over is higher than its 60-cycle spark-over, and hence its impulse ratio is greater than unity, depending on the type of gap and shape of the impulse wave. The needle gap is an extreme form of this type. Between the extremes of the uniform field gap and the non-uniform field type, as illustrated by spheres and needles, there are all the intervening forms. The horn gap so common on lightning arresters is one of these intermediate forms. The gap across line insulators from line to tower is another. Likewise a high-voltage bushing still another. When several such gaps with different characteristics are placed in parallel and subjected to the same impulses, it is clear that they will not behave alike. The fastest gap will discharge first and prevent others from flashing over; *i.e.*, the *fast* gap becomes a protective device for the *slow* gap.

To illustrate this principle, assume three gaps in parallel —a sphere gap, a horn gap and a needle gap. Suppose their spacing is such as to give the same 60-cycle spark-over. If a steep impulse is then applied, the spheres only will spark-over. If the spheres are then disconnected, the horns only will spark-over. In both cases, the needles, being slow, are protected by the faster gap. Suppose again that spheres set for 250,000 volts, 60 cycles, are paralleled with horns set for 140,000 volts, 60 cycles (all

voltage values refer to maximum peak values). A slanting wave will spark across the horns. A steep impulse will spark across the spheres, although set for 1.8 times the 60-cycle spark-over voltage of the horns. The fast spheres thus protect the slower horns, when a steep-wave-front impulse is applied. On a transmission system having many forms of air gaps in parallel, it is necessary for proper protection that the intended protective gaps should be fast and all other gaps slow. Hence the application of spheres to lightning-arrester gaps."

Oscillographs.—Oscillographs are the most important apparatus for obtaining quantitative data on electric transient phenomena. To gain clear concepts of the relative magnitude of the physical quantities involved it is highly desirable for the student to take oscillograms of a number of typical transients. For this purpose an oscillograph with a photographic attachment is necessary.

While several types of oscillographs are in commercial use all operate on the same basic principle. The essential element of oscillographs of this type, is an insulated loop of wire, placed in a magnetic field, through which the electric current flows. The direction and magnitude of the currents cause a proportional turning movement of a small mirror attached to both sides of the loop. The deflection of a beam of light thrown on the mirror indicates the angular position of the mirror and hence the magnitude and direction of the current flowing through the loop.

(*a*) *Duddell Oscillograph.*—This instrument consists essentially of a modified moving-coil galvanometer combined with a rotating or vibrating mirror, and a moving photographic film, or a falling photographic plate. The galvanometer portion, often referred to as the oscillograph, is shown diagrammatically in Fig. 10.

In a narrow gap between the poles N, S of a powerful magnet are stretched two parallel conductors s, s', formed by bending a thin strip of phosphor bronze back on itself over an ivory pulley P. A spiral spring attached to this

pulley serves to keep a uniform tension on the strips, and a guide piece G limits the length of the vibrating portion to the part actually in the magnetic field. A small mirror M bridges across the two strips as shown in the figure. Passing a current through such a vibrator causes one of the strips to advance while the other recedes, and the mirror is then turned about a vertical axis.

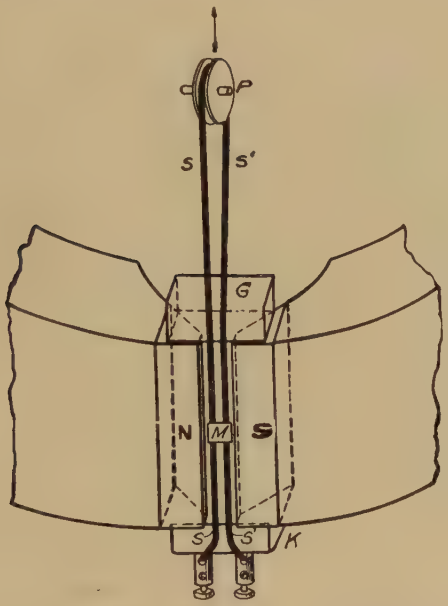

FIG. 10.—Duddell oscillograph. Vibrating element.

The whole of the vibrator, as this part of the instrument is called, is immersed in an oil bath, the object of the oil being to damp the movement of the strips, and make the instrument dead-beat.

The beam of light, coming from an arc lamp, and reflected from the mirror M, is received on a photographic film. With constant strength of the magnetic field the instantaneous value of the current is proportional to the linear displacement of the spot of light focused on the film. With alternating currents the spot of light oscillates to and fro as the current varies and with the film stationary would thus

trace a straight line. To obtain an image of the wave form, the photographic film is moved in a direction at right angles to the direction of the movement of the spot of light. Or a second mirror can be interposed in the path of the beam of light, and this mirror caused to vibrate or rotate so as to impart to the beam of light a uniform motion about an axis at right angles to the zero position of the beam and

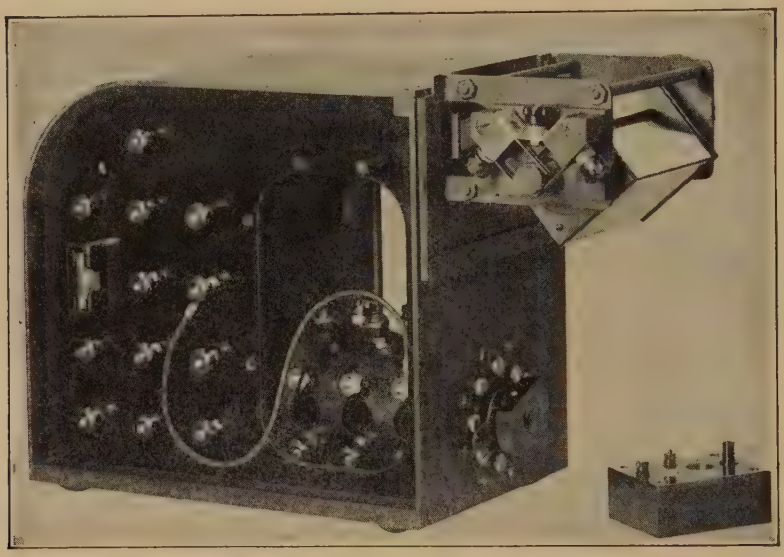

FIG. 11.—The Osiso, with motor-driven commutator and viewing mirrors attached. The permanent-magnet galvanometer is shown at the lower right. (*Westinghouse Elec. & Mfg. Co.*)

also in the initial plane of vibration. The spot of light will then trace out on a stationary screen or plate the time curve of the variation of the current flowing in the vibrator.

(*b*) *The Osiso*, Fig. 11, a single-element instrument, is extremely compact in design, the whole outfit weighing only 14 lb. The instrument proper is 6¼ in. wide, 9 in. high and 10½ in. long. It has a specially sensitive galvanometer, a 4-volt incandescent lamp with a straight-line filament, optical system, vibrator resistance and mechanism for throwing the one vibrator into six different circuits,

inserting proper resistance and shifting a cross-line to mark for identification each curve recorded on the film. The three-pole, six-position switch may be connected to six different circuits ranging in potential from 0.1 to over 220 volts. Thus, for example, by using 0.05-ohm, 5-amp. shunts in standard transformer circuits, the alternating-current waves from any six circuits may be recorded on one film. A sectional view of the Osiso is shown in Fig. 12.

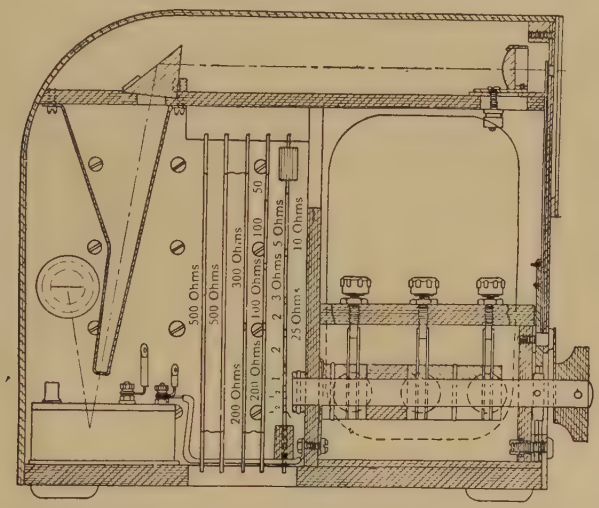

FIG. 12.—Sectional view of the Osiso. Showing the 2-watt, straight-filament lamp; horizontal galvanometer; reflecting prism; cylindrical condensing lens; rotary switch, cam-operated cross-hair; vibrator resistances and micarta case.

(*c*) *Three-element Oscillographs.*—The three-element, portable-type oscillograph manufactured by the General Electric Co. is shown in Figs. 13 to 19. The arrangement of the electromagnetic field and the three vibrating elements is shown in Fig. 13. The vertical cross-section of the vibrating elements removed from its magnetic field is shown in Fig. 14. The three vibrators are independent units and insulated so as to carry three separate electric currents. The vibrating strips and mirrors are of silver. The vibrating element can be turned around a vertical

axis, passing through the center of the mirror, by the screw Q. The containing cell for the whole vibrating element is also movable around a horizontal axis, passing through the center of the mirror, by means of the screw S. Hence the beam of light reflected from the vibrating mirror may be directed to any desired spot and so adjusted as to pass

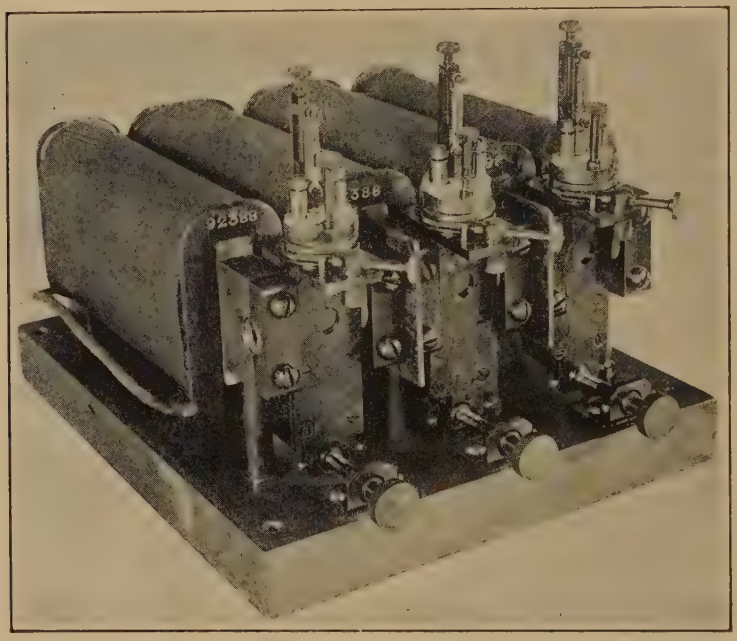

FIG. 13.—Magnetic field and vibrating elements. (*General Electric Co.*)

through the cylindrical lens to the slit in front of the rotating photographic film.

In Fig. 13, the letters TT' mark the terminals of the vibrating strips marked ST in Fig. 14. The mirror with the vibrating portion of the loop lies between the supports BB'. The size of the mirror is about 20 by 10 mils and the vibrating element has a natural period of approximately one five-thousandth of a second (0.0002 sec.). By immersing the vibrating element in oil the instrument is made dead-beat.

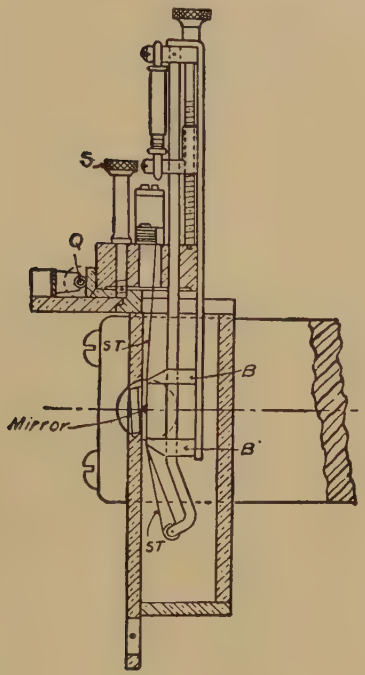

FIG. 14.—Cross-section of vibrating element.

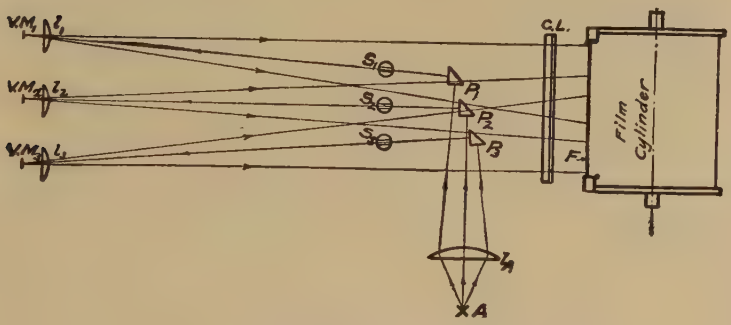

FIG. 15.—Optical train—horizontal projection.

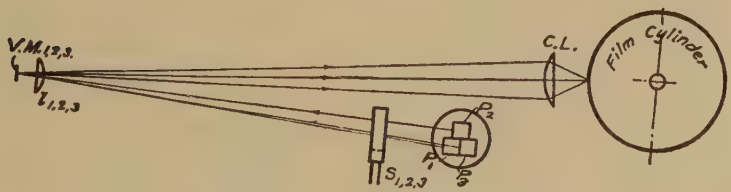

FIG. 16.—Optical train—vertical projection.

The horizontal projection of the optical train for photographic work is shown in Fig. 15 and a vertical projection in Fig. 16. The arc lamp is at A and the arrows indicate the directions of the beams of light. P_1, P_2, P_3 are right-angled prism mirrors; S_1, S_2, S_3 adjustable slits; l_1, l_2, l_3 condensing lenses; VM_1, VM_2, VM_3 the vibrating mirrors;

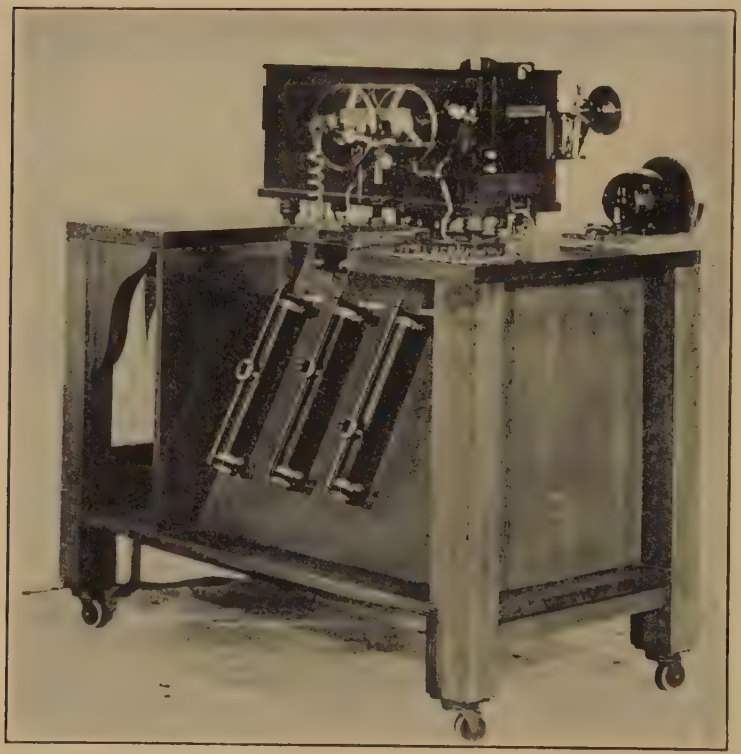

FIG. 17.—Oscillograph on operating stand. (*University of Washington.*)

CL a cylindrical lens for bringing the light beams to a sharp focus on the photographic film on the surface of the revolving cylinder in the film holder.

In Fig. 17 the oscillograph is shown mounted on a conveniently arranged operating stand. The positions of the arc lamp, film motor, film holder, controlling rheostats, time wave oscillator and other accessory appliances for

recording electric transients are clearly indicated. The corresponding wiring diagram is shown in Fig. 18.

The three-element, portable oscillograph of compact design, manufactured by the Westinghouse Elec. & Mfg. Co., is shown in Figs. 19 to 22. The photographic film drum and driving pulley with rheostats, switches, etc., are

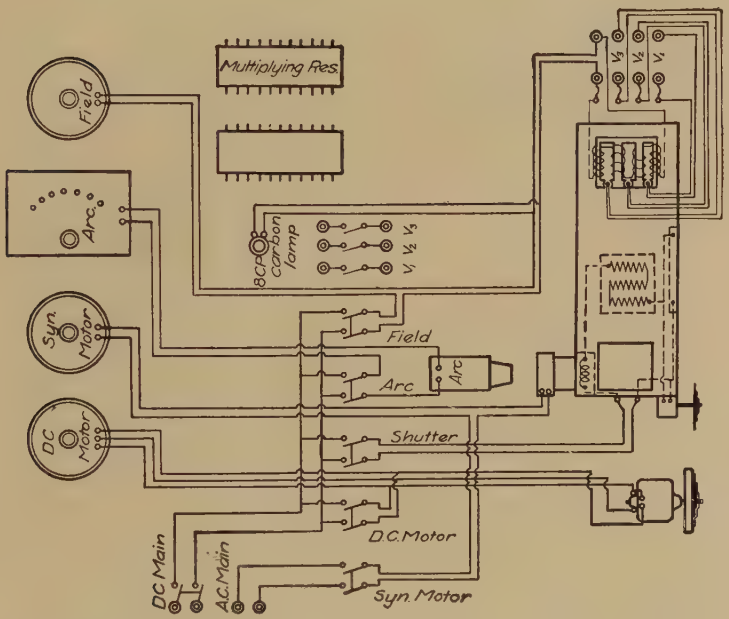

Fig. 18.—Wiring diagram—three-element oscillograph. (*General Electric Co.*)

shown on the right side of Fig. 19, while on the left are the three sets of dial resistances, one for each vibrating element, with switches, binding posts and fuses. In Fig. 20 is shown the driving motor with control apparatus for operating the film holder at several speeds. Light for making the photographic record is obtained from a low-voltage incandescent lamp of special design. For high-speed

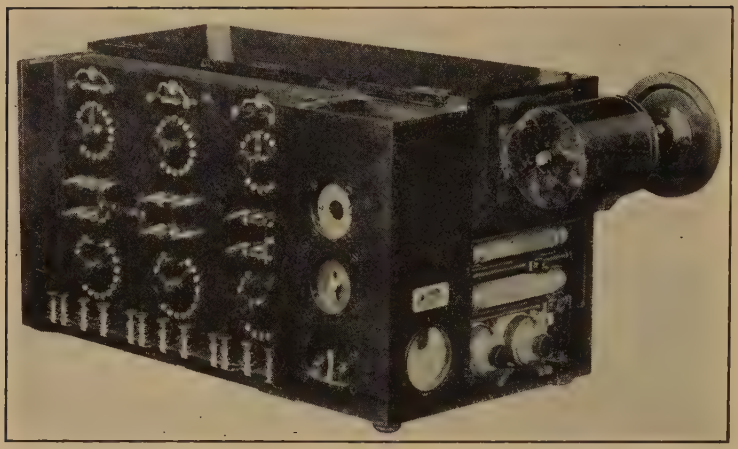

FIG. 19.—Front and resistance panel side of portable, three-element oscillograph. (*Westinghouse Elec. & Mfg. Co.*)

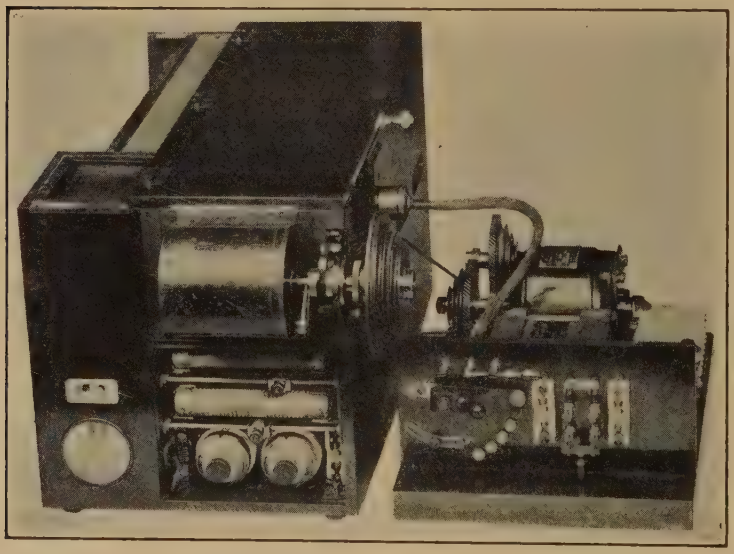

FIG. 20.—Front view of portable, three-element oscillograph coupled to motor. (*Westinghouse Elec. & Mfg. Co.*)

records an arc lamp is used, in place of the incandescent lamp, to gain the greatest possible light intensity.

The galvanometer, with one of the three elements removed, is shown in Fig. 21. The moving element consists of a single turn or oblong loop of wire forming two parallel conductors. A tiny mirror is attached to both conductors and placed in a strong magnetic field. Hence when a current passes down one conductor and up the other, one tends to move forward and the other backward. The mirror bridging these conductors is given an angular deflection proportional to the current.

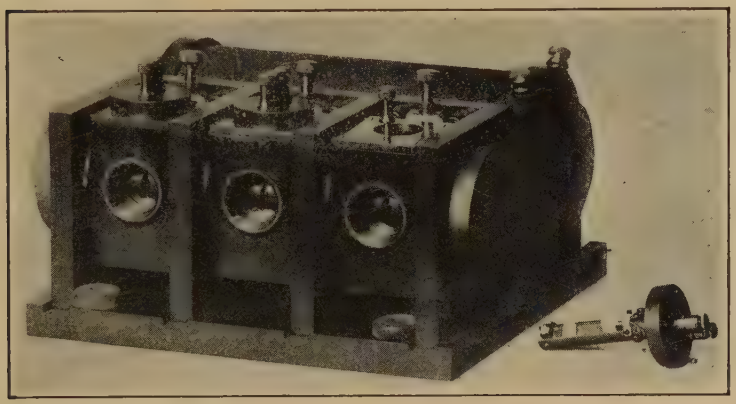

FIG. 21.—Three-element galvanometer. (*Westinghouse Elec. & Mfg. Co.*)

The design of the electromagnetic field circuit is unique. A direct current passing through a single coil sets up a magnetic flux which passes through the three vibrating elements in series. To insulate the elements from each other and from the main magnetic core and yokes four insulating gaps are used, thus placing seven air gaps in series in the path of the magnetic flux. The three gaps in the galvanometer elements are $\frac{1}{32}$ in. long, giving sufficient space for the vibrators and producing uniform distribution of the magnetic flux. The four insulating gaps are $\frac{1}{16}$ in. long but of large cross-sectional area so as to give comparatively

low reluctance in the magnetic circuit. The field excitation requires 6 volts, direct current.

A view of the trip magnet and shutter release mechanism is shown in Fig. 22, in which the trip magnet holds the long shutter finger so that the short finger does not quite touch the shutter tripping arm. The shutter is a tube with two opposite longitudinal slots. The tube rotates and when the slots are in a horizontal plane the beams of light, reflected from the tiny mirrors of the galvanometer vibrators, pass through the cylindrical condensing lens and are

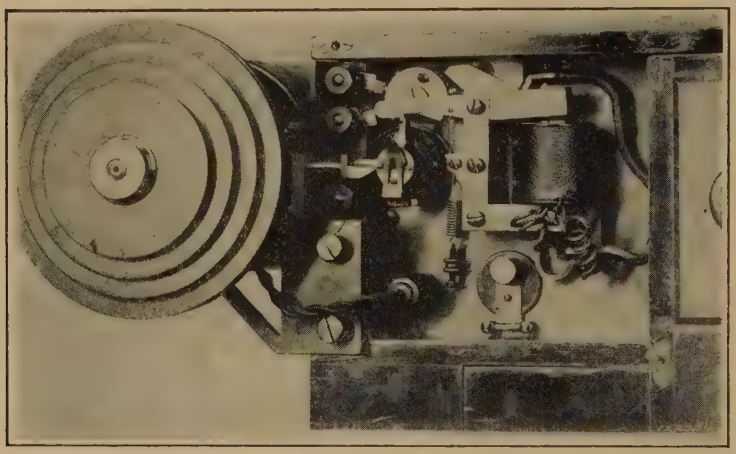

FIG. 22.—Trip magnet and shutter release mechanism. (*Westinghouse Elec. & Mfg. Co.*)

focused on the revolving photographic film. This occurs between the time the short finger falls from the shutter tripping arm and the time the variable finger falls from the arm one revolution later. The shutter is actuated by the spiral spring seen just beyond the finger hub. A pin on the shutter shaft strikes an arm on the lamp-extinguishing switch. On the hub are attached laminated copper strips which complete the lamp circuit when the shutter is set and which break the circuit when the shutter snaps closed. The tripping device can be adjusted so as to start exposures at any desired part of the film.

(*d*) *The six-element oscillograph*, manufactured by the Westinghouse Elec. & Mfg. Co., is a recent development in this field. By producing a steel having very good retaining power it became possible to use permanent-magnet galvanometers and thereby greatly reduce the size and weight of the instrument. A view of the oscillograph is

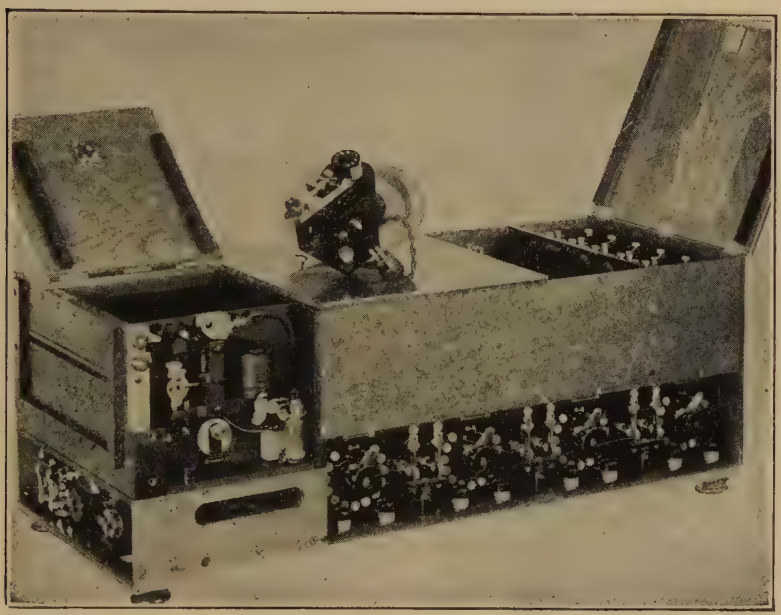

FIG. 23.—Six-element oscillograph cabinet with covers raised over optical box and galvanometer. The driving head is shown resting on top. (*Westinghouse Elec. & Mfg. Co.*)

shown in Fig. 23 and of the permanent-magnet six-element galvanometer in Fig. 24. The advantage of having six vibrators in one instrument, making it possible to record the wave form of six functions—in the systems under observation—simultaneously on the same film, is obvious.

(*e*) *Cathode-ray Oscillograph.*—The cathode-ray oscillograph tube, commonly known as the Braun tube, is an instrument that has long been used as a means for studying high-frequency electromagnetic phenomena. The advantage over other types of oscillographs lies in the fact that

the moving element consists of a beam of electrons which is practically free from inertia. The absence of inertia and of free period makes the frequency limitation of the instrument lie at least as high as the upper radio frequencies.

The beam of electrons can be deflected in any direction by applying a dielectric or a magnetic field across the beam near its source. The beam is deflected in the same direction as the dielectric field and at right angles to the direction of the magnetic field. If two fields are applied so that their

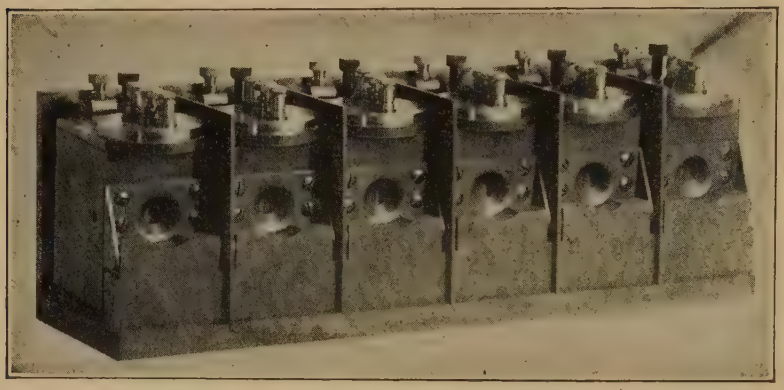

FIG. 24.—Six-element galvanometer having permanent magnets enclosed within element wall. (*Westinghouse Elec. & Mfg. Co.*)

respective deflections are at right angles, then the path traced by the end of the beam is the curve, in rectangular coördinates, of the relation between the two fields at any time. If the two fields are cyclic and synchronous, the same pattern is traced repeatedly and shows a stationary curve. In many cases these patterns are simpler to interpret and work from than patterns where the time is one of the rectangular coördinates. If one of the fields varies uniformly with the time, the curve shows the relation between the other field and the time, like the pattern shown on the rotating drum of other oscillographs. If the one field is some other known function of time, *e.g.*, sinusoidal or logarithmic, the variation of the other field

with the time can be worked out from the pattern produced. Like the galvanometer forms of oscillograph, the Braun tube is best suited for studying cyclic or recurring phenomena. Often, however, it may serve as a very convenient indicator of the approximate form of transient phenomena, if the frequency is low enough so that single sweep of the beam across the fluorescent screen can be seen or recorded photographically.

The type of oscillograph tube designed by Braun, still used occasionally in essentially the same form, derives its electron beam from a high-voltage electric discharge. The structure of the tube is shown in Fig. 25, in which A is the anode, C the cathode, D a diaphragm with a small hole, P_x, P_y deflecting plates and S the fluorescent screen.

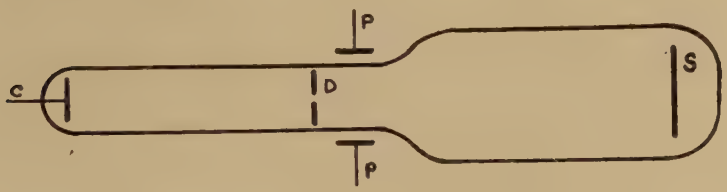

FIG. 25.—Diagram of Braun-tube oscillograph.

Direct-current voltage from 10,000 to 50,000 volts is used to operate these tubes, requiring an installation which is expensive, not easily portable and dangerous. Because of the high-voltage discharge the gas is rapidly absorbed on the walls of the tube and means must be provided for letting more gas into the tube from time to time, and for pumping it out when too much gas is admitted. These undesirable features have kept the Braun tube from coming into the general use it otherwise deserves.

The tube used in the cathode-ray oscillograph differs from the original Braun tube in that the source of electron is a hot filament instead of a gas discharge. This tube operates at a much lower potential, 250 to 400 volts, with simpler auxiliary apparatus and has greater sensitiveness.

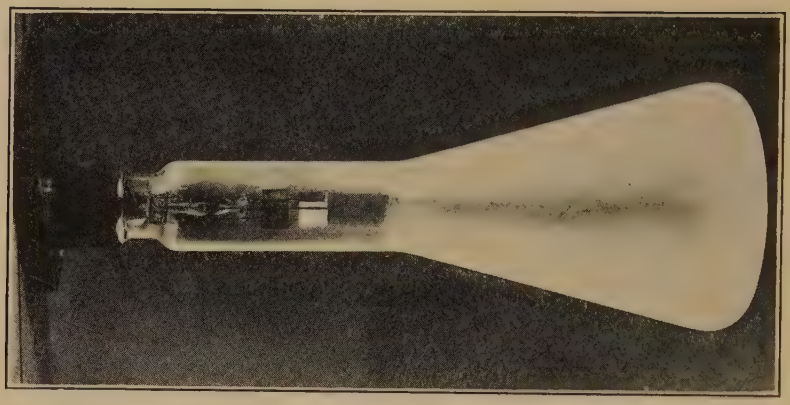

FIG. 26.—Cathode-ray oscillograph. Hot filament cathode type.

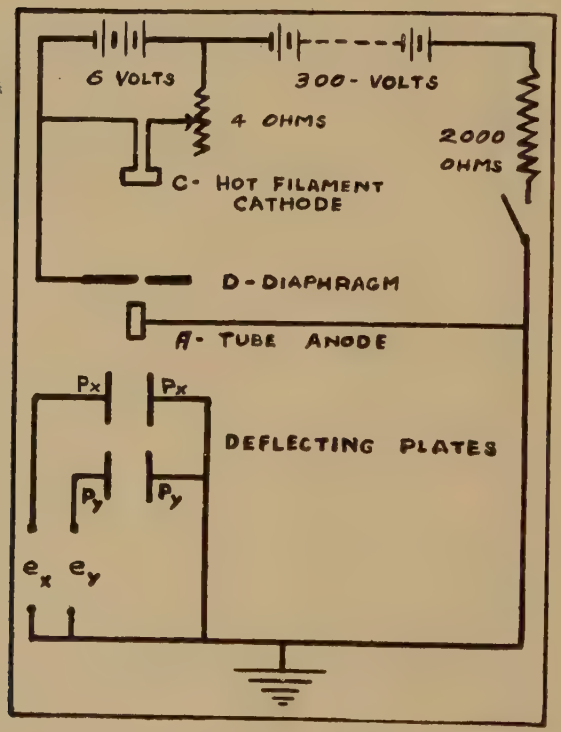

FIG. 27.—Circuit diagram. Cathode-ray oscillograph.

A laboratory type of tube, manufactured by the Western Electric Co., used in the cathode-ray oscillograph is shown in Fig. 26, the corresponding circuit diagram in Fig. 27 and a perspective operation diagram in Fig. 28.

The electrons are projected from the hot-filament cathode through the small opening in the diaphragm and the tube-shaped anode, thus forming a well-defined beam or ray which passes between the deflector plates P_x, P_y and impinges on the fluorescent screen, the large end of the tube.

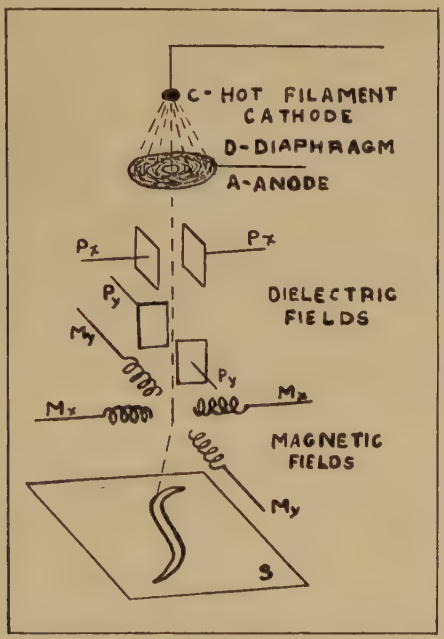

FIG. 28.—Perspective operation diagram. Cathode-ray oscillograph.

Two pairs of deflector plates are mounted near the anode at right angles to each other, one plate of each pair being connected to the anode.

The potentials to be studied, e_x and e_y, are applied between the anode and the other plate of each pair of deflectors. In order to deflect the electron beam by magnetic fields, two pairs of coils are placed at right angles

to the tube and spaced 90°, as indicated by M_x and M_y in Fig. 28. The large end of the containing tube is covered with a fluorescent material which becomes luminous at the spot where the electrons strike it. The tube contains a small amount of gas which serves the double purpose of focusing the beam of electrons and preventing charges from accumulating on the walls of the tube.

(*f*) *The Dufour Cathode-ray Oscillograph.*—In the Dufour[1] type of cathode-ray oscillograph the sensitized films or plates are placed inside the vacuum tube, thus permitting the full intensity of the oscillating beam of cathode rays to reach the photographic plate. By sweeping the cathode beam across the sensitized plate by means of a magnetic field at right angles to the direction of the timing wave oscillations, frequencies greater than 150,000,000 cycles per second have been recorded.

Timing Waves from the Oscillator Alternator.—The time factor is of special importance in electric transient phenomena and some means for recording the time elapsed is necessary. In taking oscillograms in which the transient current or voltage recorded does not give directly an indication of the time consumed, it is customary to impress an alternating-current timing wave of known frequency on one of the vibrators.

Current for the timing wave may be taken directly from any available power circuit, but the frequencies of commercial systems are to some extent variable and the indicating frequency meters may not be sufficiently accurate for this purpose.

(*g*) *Oscillator Generator.*—A convenient source for obtaining constant-frequency timing-wave currents for oscillograms is found in the oscillator generator. A simple portable form that has proved satisfactory in the electrical engineering laboratories of the University of Washington is shown in Fig. 29.

[1] Dufour, A., "Oscillographe Cathodique," *Bull.* École Supérieure D'Électricité, Paris.

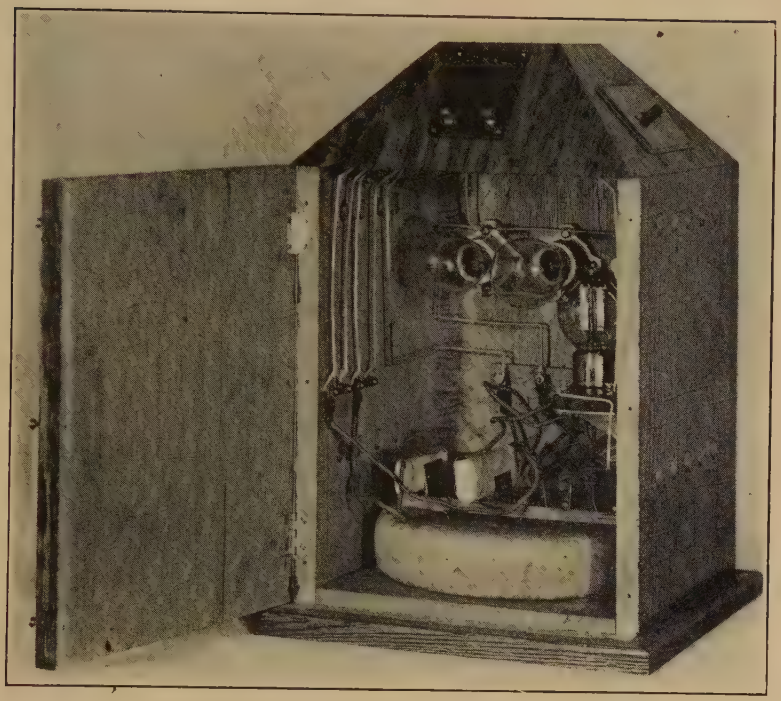

FIG. 29.—Oscillator alternator. (*University of Washington.*)

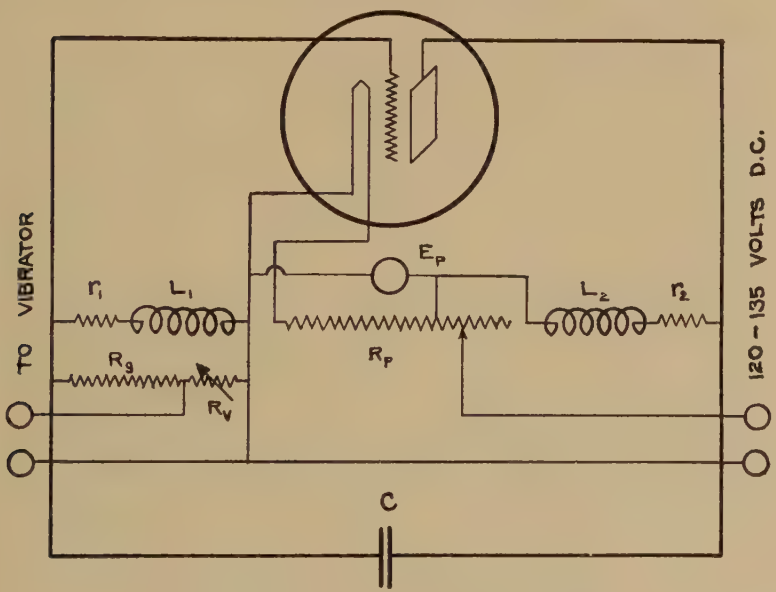

FIG. 30.—Circuit diagram of oscillator alternator.

The alternator consists of a triode tube connected to condensance, inductance and resistance arranged as indicated by the circuit diagram in Fig. 30. The apparatus is designed to generate alternating current at a frequency of either 100 or 200 cycles per second when 120 volts of direct current are applied to the supply terminals. The following quantitative values for the circuit constants indicated in Fig. 30 will give constant frequency and amplitude suitable for timing waves on oscillograms.

For 100 cycles per second:

L_1 = 0.515 henry; L_2 = 1.045 henrys;
L_1L_2 (series additive) = 3.08 henrys;
C = 0.805 mf.; R_g = 1,500 ohms; R_v = 30 ohms;
r_1 = 69.4 ohms; r_2 = 104.7 ohms;
R_p = 110 ohms constant, 15 ohms variable, 170 watts radiating capacity.
Tube; Western Electric V.T.–II
E_p = 120 volts; I_f = 1.20 amp.

The amplitude of the wave may be altered by varying the resistance R_4; likewise more or less than 120 volts direct-current voltage may be used by adjusting the resistance R_3. After the circuit constants have been adjusted so that the alternator will produce currents of the desired frequency and wave amplitude the oscillator generator becomes a convenient and dependable device for securing a time measure directly on the film when taking oscillograms of electric transients.

(*h*) *Oscillograms.*—Careful, painstaking work is required in order to secure good oscillograms. Every transient to be recorded becomes a separate problem. The speed of the revolving drum carrying the sensitized paper or film, the amplitude of the galvanometer vibration, the

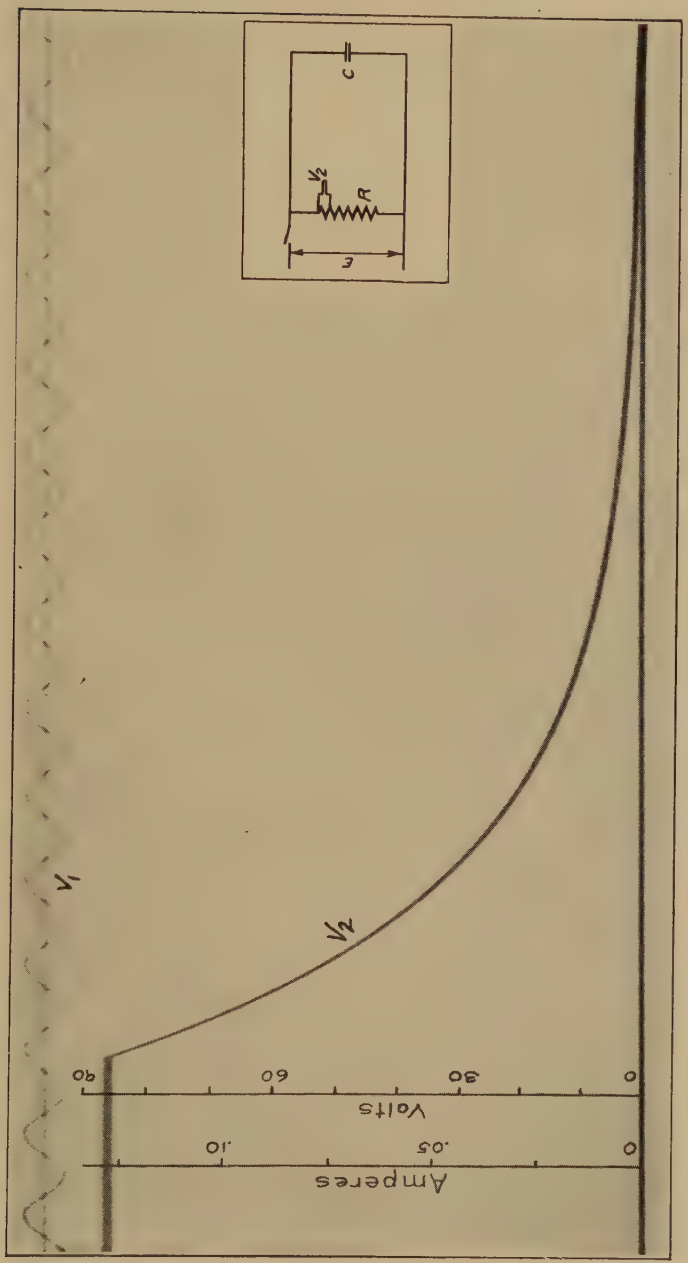

FIG. 31.—Discharging a condenser through a constant resistance.
E = 86 volts; c = 35 mf.; R = 677 ohms; I = 0.127 amp.; v_1 = 100~.

sharpness and size of the spot of light producing the photographic record, the time lag of tripping devices and switches and many other factors must be adjusted in each case so as to comply with the conditions imposed by the transient under investigation. Detailed instructions for developing and printing the oscillograms are given in Chap. X.

Figures 31 to 38 show good oscillograms of typical laboratory electric transients illustrating the factors men-

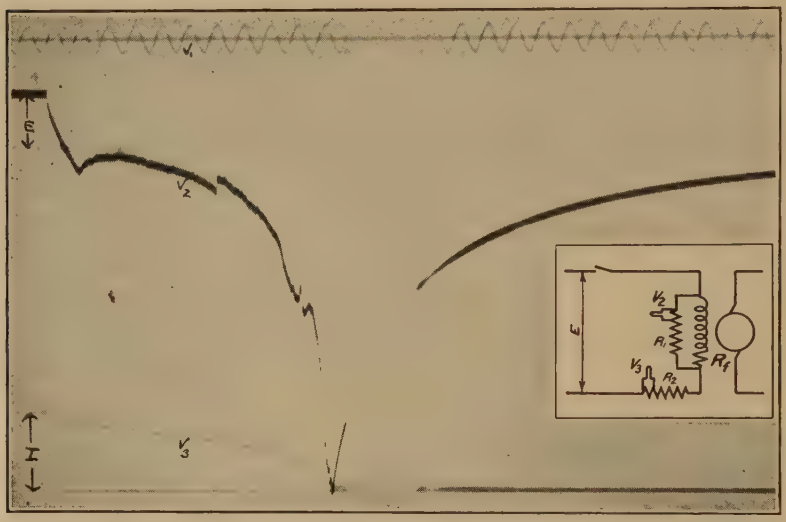

FIG. 32.—Breaking field circuit of a direct-current motor.
E = 74.5 volts; I = 1.5 amp.; R_1 = 1,090 ohms; R_f = 49.6 ohms; v_1 = 100~

tioned above. The circuit diagram on each oscillogram indicates the circuit condition and shows the specific points at which the vibrators recording the transients were inserted. As an indication of the changes in adjustments on the oscillograph, it may be noted that the relative drum speeds for the oscillograms shown in Figs. 31, 32, 34 and 38 were as 20 : 7.3 : 1 : 77, while the amplitude of the vibration was in each case adjusted so as to use the film to good advantage.

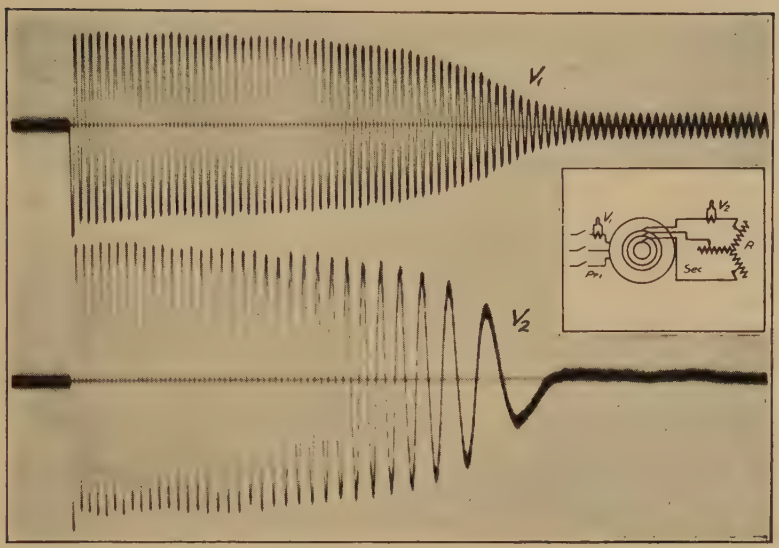

FIG. 33.—Starting current in an induction motor with wound rotor. E = 95.5 volts; $I_{pri.}$ = 14 amp. (running); $I_{sec.}$ = 95 amp. (starting); R = 0; f = 60~.

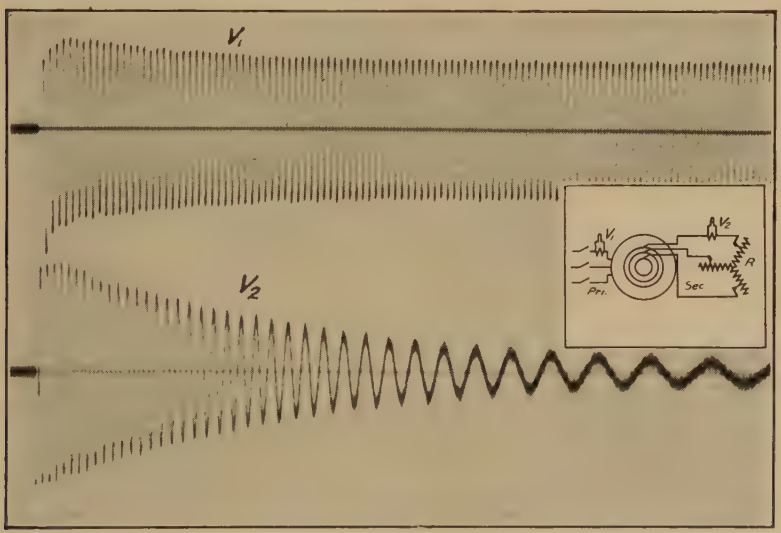

FIG. 34.—Starting current in an induction motor with wound rotor. E = 159 volts; $I_{pri.}$ = 43.5 amp. (running); $I_{sec.}$ = 22 amp. (starting max.); R all in; f = 60~.

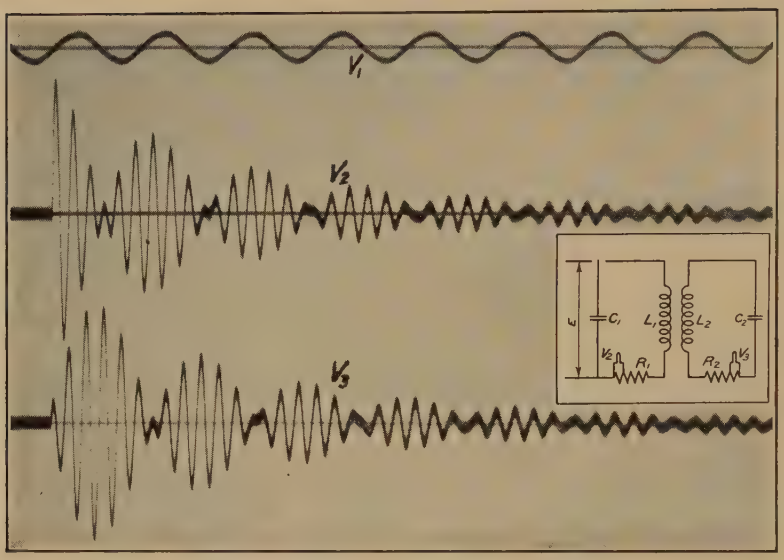

FIG. 35.—Energy transfers in inductively coupled circuits.

E = 375 volts; L_1 = 0.237 henry; C_1 = 0.45 mf.; R_1 = 12.6 ohms; $v_1 = 100\sim$; L_2 = 0.237 henry; C_2 = 0.45 mf ; R_2 = 13.0 ohms; coupling coefficient, 14 per cent.

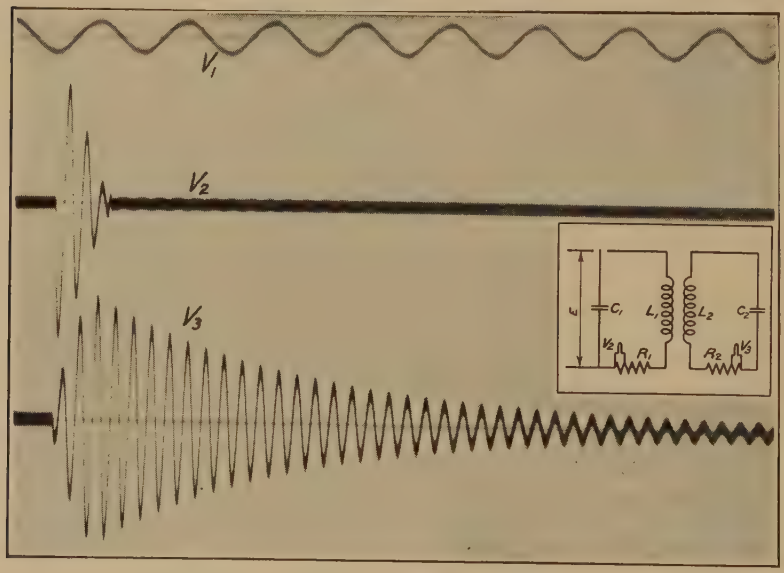

FIG. 36.—Coupled circuits. Primary circuit opened after first transfer of the energy to the secondary circuit.

E = 375 volts; L_1 = 0.237 henry; C_1 = 0.45 mf.; R_1 = 12.6 ohms; $v_1 = 100\sim$; L_2 = 0.237 henry; C_2 = 0.45 mf.; R_2 = 13.0 ohms; coupling coefficient, 14 per cent.

Figure 31 shows the transient current-time or voltage-time curves in discharging a condenser through a constant resistance. It is evident that the discharge is not instantaneous but requires measurable time and that the rate of discharge is greatest at the instant the switch is closed and then rapidly decreases with the decrease of energy stored in the condenser.

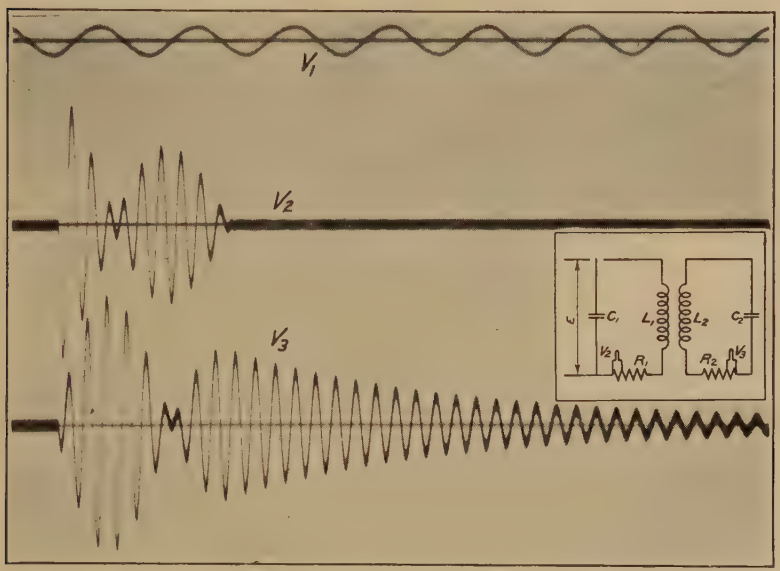

FIG. 37.—Coupled circuits. Primary circuit opened after the second transfer of the energy to the secondary circuit.

E = 375 volts; L_1 = 0.237 henry; C_1 = 0.45 mf.; R_1 = 12.6 ohms; v_1 = 100~; L_2 = 0.237 henry; C_2 = 0.45 mf.; R_2 = 13.0 ohms; coupling coefficient, 14 per cent.

The voltage and current transients produced by the opening of the field circuit of a direct-current motor are shown in Fig. 32. Note that, while only 74.5 volts from the direct-current supply were impressed on the motor field, a transient voltage of 530 volts was produced at the field terminals when the circuit was broken.

The starting currents for both the stator and rotor windings of an induction motor are shown in Figs. 33 and

34. Note the difference in the current and time required to bring the rotor up to operating speed with the external resistance "in" or "out" of the secondary current.

Energy transfers in inductively coupled circuits are shown in Figs. 35, 36 and 37. The electric energy impressed on the primary is transferred inductively to the secondary. With both circuits closed as shown in Fig. 35, the energy is transferred to and fro between the two circuits with the successive oscillations, or beats, decreasing in amplitude until all the electric energy has been transformed into heat by the Ri^2 losses. By opening the primary circuit after the first transfer as in Fig. 36 or at the end of the second period as in Fig. 37, the oscillations are then confined to the secondary circuit, as no further transfers are possible. The exactness with which the sequence of events may be controlled by the operator is illustrated by the opening of the primary circuit in Figs. 36 and 37. By referring to the time wave v_1 it appears that in both cases the primary was opened within one ten-thousandth of a second of the instant predetermined by the operator. During each transfer part of the electric energy appearing alternately in the primary and secondary circuits is dissipated into heat and hence the successive resonant oscillations decrease in magnitude.

Standing waves or free oscillations of transmission lines are illustrated by the oscillogram in Fig. 38. From the circuit diagram and the given quantitative data it is evident that the length of the line represents one-fourth wave length of the fundamental frequency recorded on the oscillogram.

The Klydonograph.—The klydonograph is a voltage recorder invented by J. F. Peters and developed by the Westinghouse Elec. & Mfg. Co. The circuit diagram in Fig. 39 shows the essential elements of the instrument in its simplest form. Lead A, Fig. 39, is connected to the source of voltage to be recorded, usually through some form of electrostatic potentiometer, and lead B is grounded.

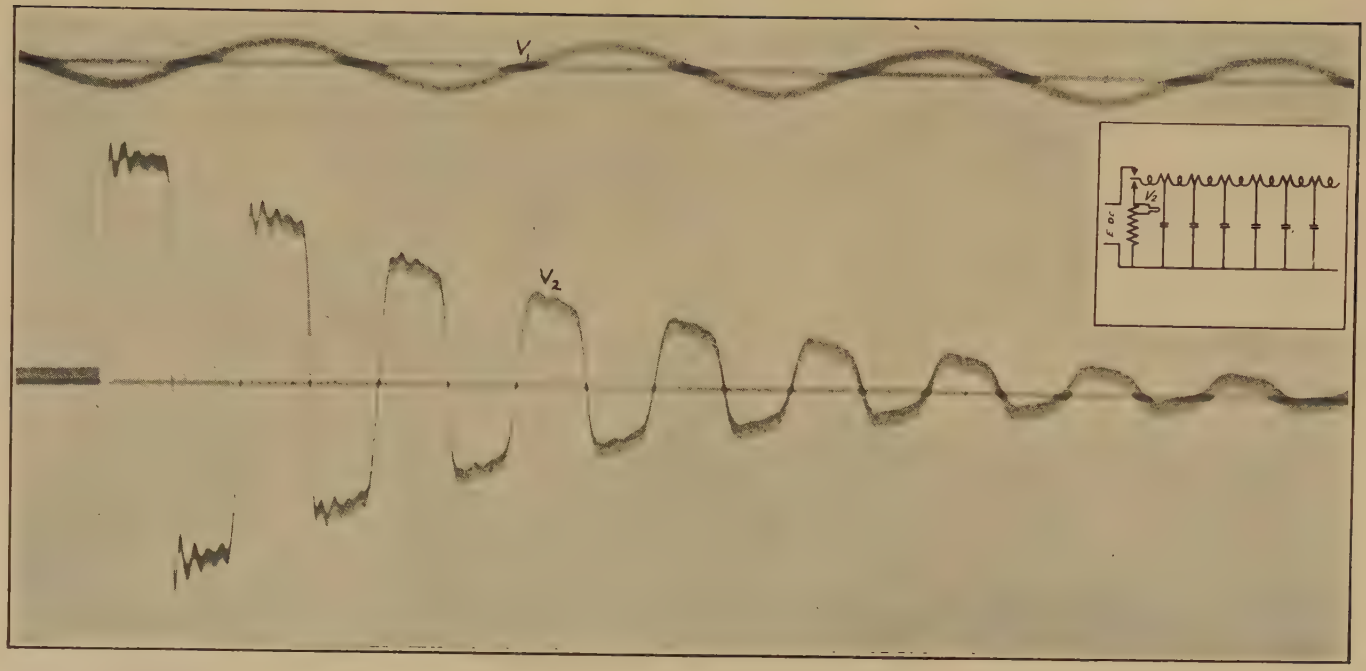

FIG. 38.—Oscillation of a 200-mile artificial transmission line. Lumpy type. Receiver end open. Vibrator v_2 at generator end of line. $E = 300$ volts; $R = 49.4$ ohms; $L = 0.388$ henry; $C = 3.06$ mf., $v_1 = 100\sim$.

By impressing a voltage E greater than the threshold or critical voltage (about 2 kv.) on the leads of the instrument, the photographic plate will be affected, so that, when developed, a figure will appear surrounding the spot where

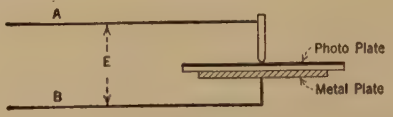

FIG. 39.—Circuit diagram of the klydonograph.

the terminal of A had been in contact with the sensitized surface. Different wave forms will produce so called Lichtenberg figures of different patterns, so that the photographic record, or klydonogram, gives information in regard to the

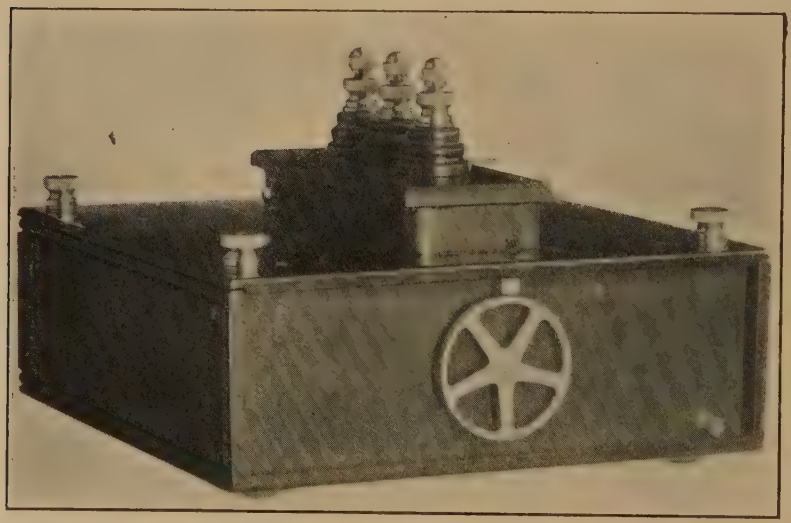

FIG. 40.—Three-element klydonograph. (*Westinghouse Elec. & Mfg. Co.*)

magnitude, wave front and polarity of the impressed voltage and whether or not the surge was oscillatory.

In construction, the klydonograph is a simple instrument, essentially merely a metallic point in contact with a photographic plate. A three-element klydonograph is shown in Fig. 40, which records three sets of data simultaneously on the same film. Examples of Lichtenberg

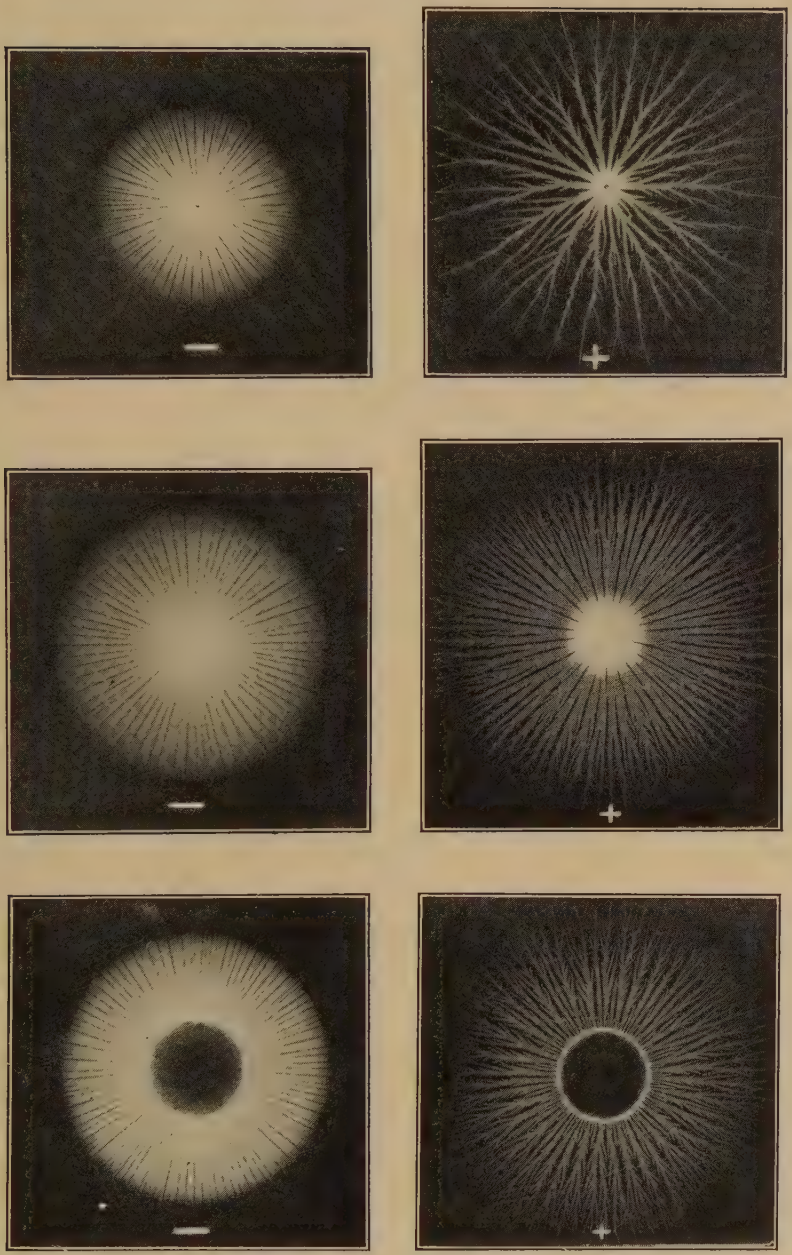

FIG. 41.—Lichtenberg Figures.

figures which form the basis for the klydonograph and similar surge recorders are shown in Fig. 41. Various types Lichtenberg figures are shown in Figs. 210, 211 and 212.

Problems and Experiments

1. Examine the oscillograph with care; trace all the circuits; operate the arc lamp; adjust the optical train until the mirror on each vibrator throws a spot of light through the slit and this is sharply focused on the ground-glass screen. Arrange a circuit with variable inductance and condensance as indicated in Fig. 42. Connect vibrator v_1 by means of a shunt S so as to indicate the current wave and vibrator v_2 with a resistance R_1 in

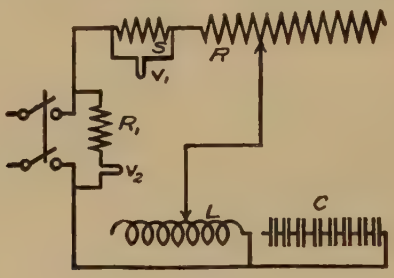

FIG. 42.

series to show the voltage wave. By means of the small synchronous motor operate the large oscillating mirror throwing the beams of light on the mica screen on top of the oscillograph. By varying the resistance, inductance and condensance in the circuit the time-phase relations of the voltage and current may be changed from lag to lead.

2. (*a*) What fraction of the energy initially stored in the condenser in Fig. 31 still remains when the voltage across the terminals has dropped to 50 volts? 10 volts? ½ volt?

(*b*) From the oscillogram, Fig. 31, find the time required to dissipate into heat half of the energy initially stored in the condenser. Three-fourths. Nine-tenths.

3. An oscillograph was connected to measure the overload current of an alternating-current circuit (sinusoidal wave form) as shown in circuit diagram, Fig. 43. Resistance of each vibrator circuit is 11.5 ohms. Resistance R in series with each vibrator is 10 ohms; the 5-amp. shunt S has a resistance of 0.1 ohm; the ratio of the current transformer, *C.T.*, is 40: 1.

The total peak-to-peak deflection obtained on the oscillograph was 57 mm. The vibrator of the oscillograph was calibrated by disconnecting the leads from the shunt S and connecting them across a dry cell giving 1.2 volts. The deflection was 19 mm.

Determine:

(*a*) The calibration for the vibrator in amperes per millimeter.

(*b*) The effective, or r.m.s., value of the alternating current I in the A-C line as measured on the oscillograph.

4. (*a*) Why is the transient voltage produced by breaking the field circuit of the motor in Fig. 32 greater than the operating voltage from the mains?

(*b*) What effect would a quick, as compared to a slow, opening of the switch, Fig. 32, have on the transient voltage across the field terminals?

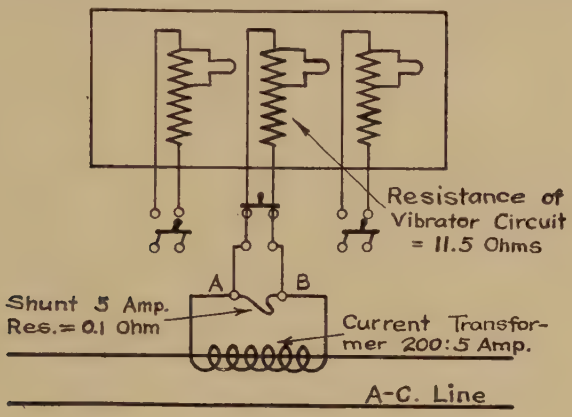

FIG. 43.

(*c*) From the oscillogram, Fig. 32, determine the fraction of a second during which the switch was arcing.

5. Reproduce in the laboratory the oscillogram shown in Fig. 32.

6. (*a*) Find the frequency of oscillation of the current in both the primary and secondary circuits in Figs. 35, 36 and 37.

(*b*) Find the frequency of energy transfer between the two circuits in Fig. 35.

7. Find the time of exposure represented by the parts of the oscillogram shown in Figs. 31, 33, 34, 35 and 36.

CHAPTER III

SINGLE-ENERGY TRANSIENTS. DIRECT CURRENTS

Transient electric phenomena are produced by changes in the magnitude, distribution and form of the energy stored in electric systems. The simplest types of electric transients are found in electric circuits having only one kind of energy storage—that is, either the magnetic or the dielectric field, but not both.

A condenser discharging through a constant non-inductive resistance, as illustrated in Fig. 44, or a magnetic field likewise discharging through a constant resistance, as shown in Fig. 45, gives transients of the simplest type. Since the resistance in the circuit, Fig. 44 or 45, is constant, the current is at all instants directly proportional to the voltage across the terminals, and therefore the curve traced on the oscillogram represents either current-time or voltage-time relations during the discharge. Both current and voltage scales appear on the oscillograms, from which the quantitative values may be obtained for any instant of time.

The energy stored in the condenser, Fig. 44, is at any instant equal to $Ce^2/2$, and the rate of discharge ei must be equal to the Ri^2 rate of energy dissipation into heat. Since the voltage e across the terminals of the condenser is directly proportional to the current i, the rate of energy discharge must at any instant be proportional to the energy in the condenser, and hence: *The rate of change in the current is at any instant directly proportional to the magnitude of the current* and likewise *the rate of change in the voltage is at any instant directly proportional to the magnitude of the voltage.*

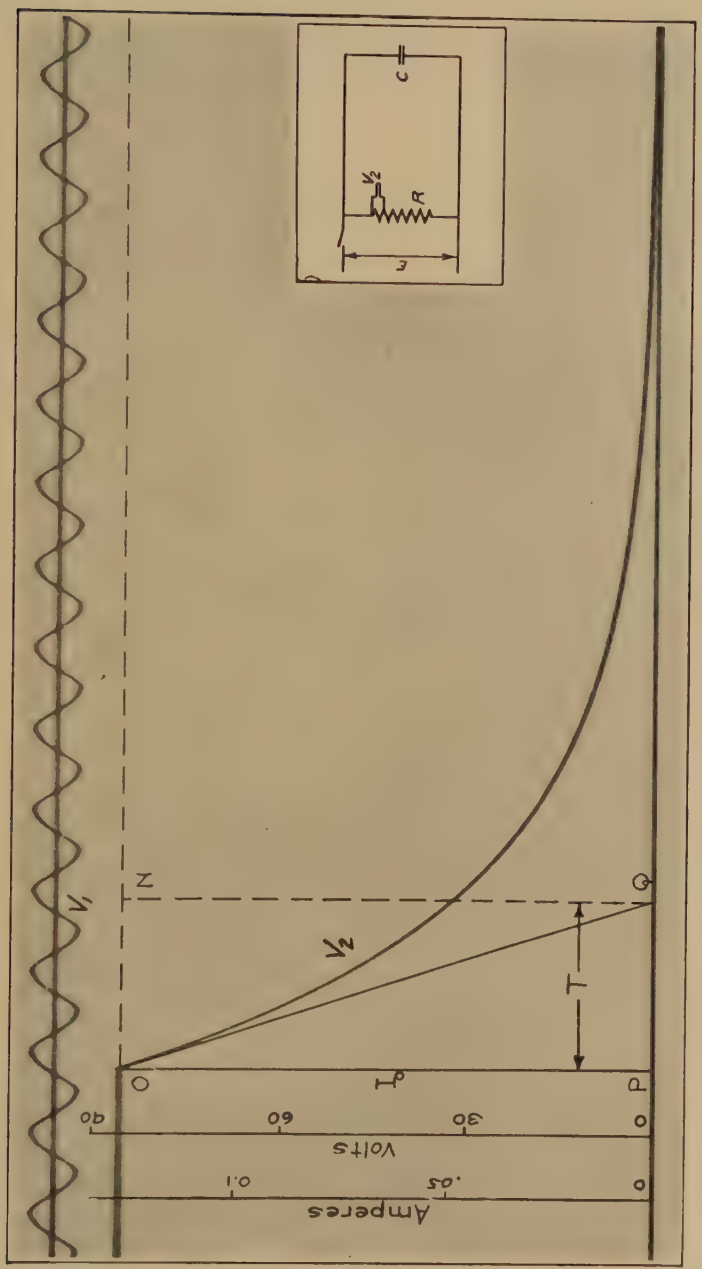

FIG. 44.—Discharging a condenser through a constant resistance.
$E = 86$ volts; $C = 30.7$ mf.; $R = 677$ ohms; $I_0 = 0.127$ amp.; $v_1 = 100$.

If i and i' represent the currents at any two points on the current-time curve in Fig. 45, the above-stated relation may be expressed by equation (30).

$$\frac{di}{dt} : \frac{di'}{dt} :: i : i' \tag{30}$$

This simple relation is of fundamental importance and may well be considered as the *basic law of transient electric phenomena,* although it can readily be derived from Ohm's and Kirchhoff's laws. Let ${}_{L}e$ be the voltage produced by self-induction and ${}_{R}e$ the corresponding voltage across the resistance. Then for any two points on the current-time curve in Fig. 45 we have from Kirchhoff's laws:

$${}_{R}e + {}_{L}e = 0, \quad {}_{R}e' + {}_{L}e' = 0 \tag{31}$$

Likewise from Ohm's law:

$${}_{R}e = Ri; \quad {}_{R}e' = Ri' \tag{32}$$

$${}_{L}e = L\frac{di}{dt}; \quad {}_{L}e' = L\frac{di'}{dt} \tag{33}$$

Hence

$$L\frac{di}{dt} : L\frac{di'}{dt} :: Ri : Ri' \text{ or } \frac{di}{dt} : \frac{di'}{dt} :: i : i' \tag{34}$$

In Figs. 44 and 45, let the line OP be drawn from starting point O perpendicular to the X-axis. Let the line OQ be drawn tangent to the curve at O and intersecting the X-axis at Q. The time represented by the distance PQ is called the time constant T of the circuit. Since i' may be any point on the curve, let it be taken at the starting point O; then

$$i' = I_0 \text{ and } \frac{di'}{dt} = -\frac{I_0}{T} \tag{35}$$

From equations (34) and (35)

$$\frac{di}{dt} : -\frac{I_0}{T} :: i : I_0 \tag{36}$$

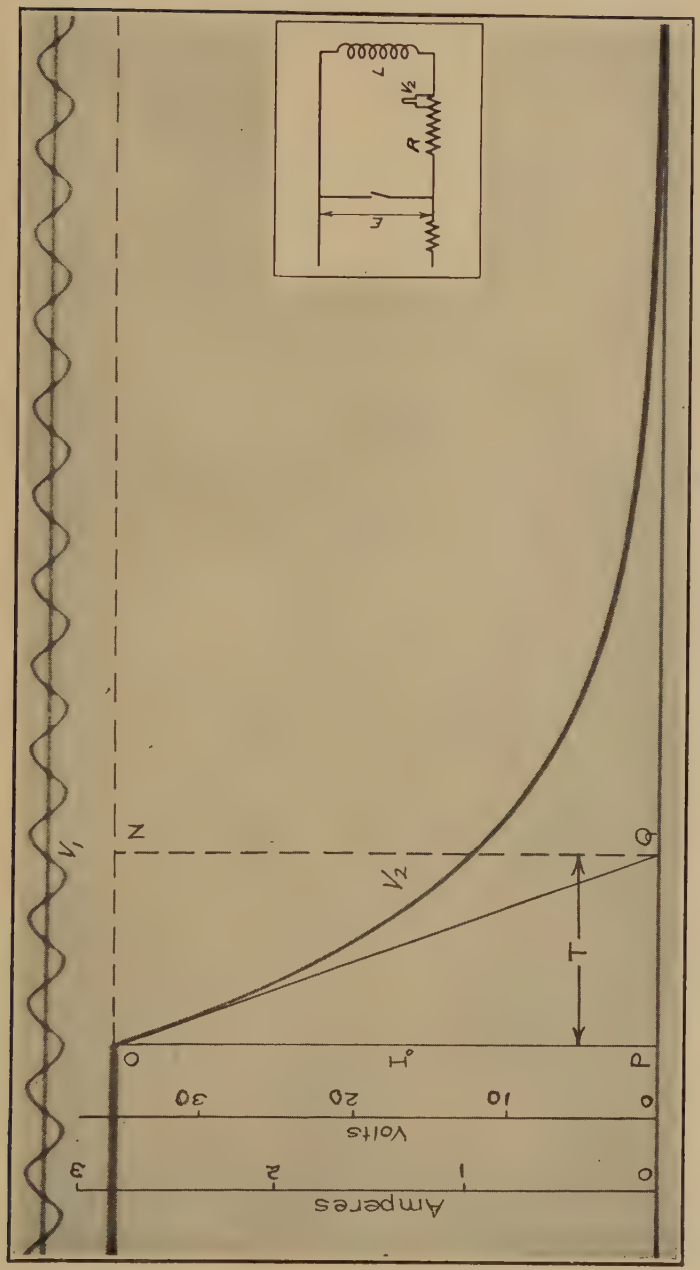

FIG. 45.—Discharging a magnetic field through a constant resistance.
E = 36.0 volts; L = 0.286 henry; R = 12.7 ohms; I_0 = 2.85 amp.; v_1 = 100~.

Separating the variables and taking the limits of integration from the starting point O to any point (i, t) on the curve:

$$\int_{I_0}^{i} \frac{di}{i} = -\frac{1}{T}\int_{o}^{t} dt \tag{37}$$

$$\log_\epsilon \frac{i}{I_0} = -\frac{t}{T} \tag{38}$$

$$i = I_0 \epsilon^{-\frac{t}{T}} \tag{39}$$

Similarly, for the corresponding voltage-time curve:

$$e = E_0 \epsilon^{-\frac{t}{T}} \tag{40}$$

Equations (39) and (40) show that *the fundamental relations in simple electric transients are expressed by the exponential equation.* The minus sign is used as di/dt is negative. The exponential curve represented by equations (39) and (40) is as fundamental in the study of electric transients as the sine wave in alternating currents.

For the same reason and in the same manner the electric transients existing while a condenser is charged, Fig. 48, or while a magnetic field is established, Fig. 46, may be expressed by the exponential equation. In the upper half of Fig. 46 is shown the oscillogram of a current-time transient obtained while establishing a magnetic field in the circuit indicated in the diagram. Let the line OP be drawn through the starting point O at right angles to the X-axis. Let PU be drawn parallel to the X-axis and be an asymptote to the current-time curve. Let the line OQ be drawn tangent to the curve at O and intersecting the line PU at Q. The length of the line OP represents the value of the permanent current I which is equal to E/R. The time measured by the line PQ is the time constant T of the circuit. The rate of storing the magnetic energy at any instant is proportional to the remaining magnetic storage facilities in the circuit under the given conditions. Therefore the rate of change in the current is at any point on

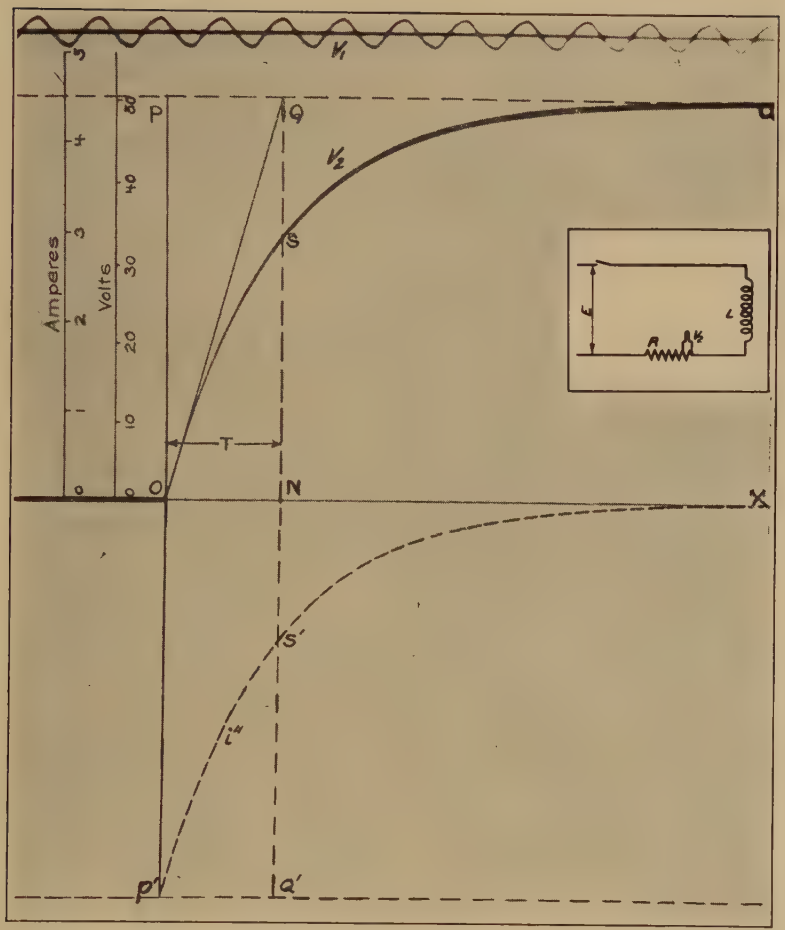

FIG. 46.—Forming a magnetic field. Air-core circuit.
E = 50.5 volts; L = 0.209 henry; R = 11.2 ohms; I = 4.5 amp.; v_1 = 100~.

the curve proportional to $I - i$, and equation (41) is derived in the same manner as equations (34) and (36)

$$\frac{d(I-i)}{dt} : -\frac{I}{T} :: (I-i) : I \tag{41}$$

$$\int_o^i \frac{d(i-I)}{i-I} = -\frac{1}{T}\int_o^t dt \tag{42}$$

$$\log_\epsilon\left(\frac{i-I}{(-I)}\right) = -\frac{t}{T} \tag{43}$$

$$i = I - I\epsilon^{-\frac{t}{T}} \tag{44}$$

The transient which by definition represents the change from one permanent condition to another is in equation (44) represented by the factor $-I\epsilon^{-\frac{t}{T}}$. Before the circuit was closed the value of the current was zero, while the final permanent value is I. Hence the initial value of the current transient was $-I$, and the corresponding voltage transient $-E$.

In Fig. 46 the initial value of the transient current is represented by OP', equal in length to OP. The transient values are given by the ordinates to the broken curve $P'S'$, while the instantaneous current which must at any instant be equal to the algebraic sum of the permanent and transient values is given by the ordinates to the curve OSU, which is the curve photographed on the oscillogram in Fig. 46. It is important to note that the photographic record of the actual instantaneous values gives at each point the resultant or the algebraic sum of the corresponding permanent and transient ordinates. Thus for any time t:

$$NS = NQ + (-NS') \tag{45}$$

that is, *the actual value of the variable (current) is at any instant equal to the algebraic sum of the permanent value and the transient value at the given instant.*

This statement applies to circuits in which the inductance, resistance and condensance are constant during the transient period. In iron-clad circuits, as illustrated in forming a magnetic field in Fig. 47, the inductance varies with the flux density and the current-time relation depends also on the permeability of the iron.

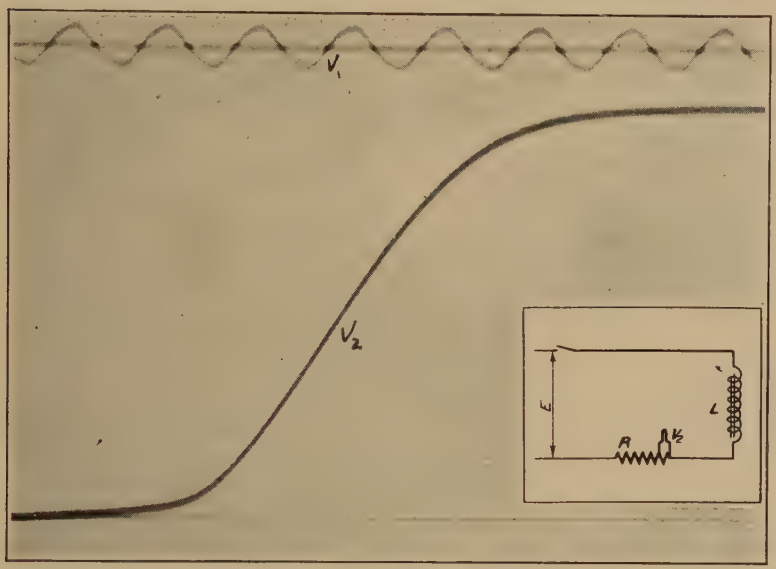

FIG. 47.—Forming a magnet field. Iron-clad circuit.
E = 21 volts direct current; I = 6.5 amp.; R = 3.2 ohms; secondary circuit of a 3-kv.a. transformer. $v_1 = 100\sim$.

The Time Constant.—From the starting point O of the current-time curve in the oscillogram, Fig. 45, which shows the discharge of a magnetic field through a resistance, the line OP is drawn at right angles to the X-axis; the line OQ tangent to the curve at the point O and intersecting the X-axis at Q; the line QN perpendicular and ON parallel to the X-axis.

From the principle of the conservation of energy, the energy stored in the magnetic field must be equal to the amount dissipated as heat in the resistance of the circuit when the field is discharged.

$$\frac{LI^2}{2} = R\int_o^\infty i^2dt = RI_0^2\int_o^\infty \epsilon^{-\frac{2t}{T}}dt = \frac{RI_0^2T}{2} \qquad (46)$$

$$T = \frac{L}{R} \qquad (47)$$

In circuits having resistance and inductance in series, as in Fig. 45, the time constant is equal to the inductance divided by the resistance.

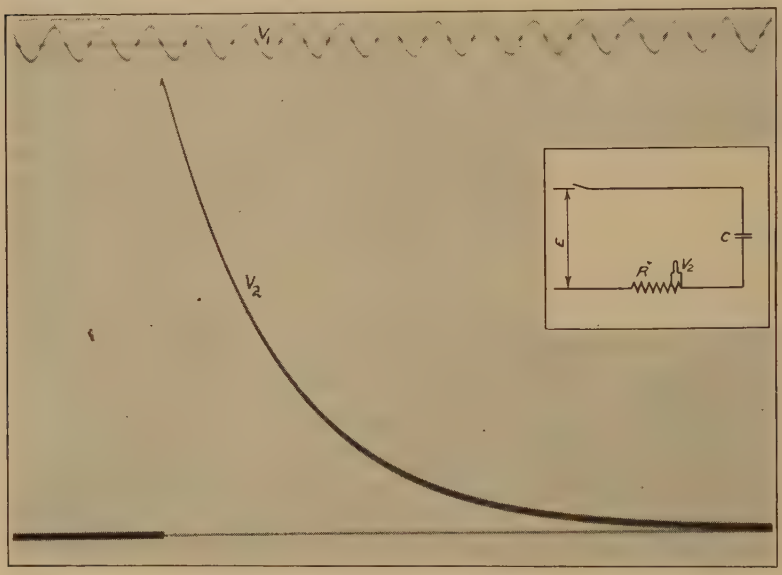

FIG. 48.—Charging a condenser.
E = 103 volts; C = 34.5 mf.; R = 667 ohms; v_1 = 100∼.

In a similar manner the expression for the time constant T in terms of the circuit constants may be found for circuits having condensance and conductance, Fig. 44. The energy stored in the condenser when the discharge starts must be equal to the energy expended as heat in the Ri^2 losses.

$$\frac{CE_0^2}{2} = R\int_o^\infty i^2dt = RI_0^2\int_o^\infty \epsilon^{-\frac{2t}{T}}dt = \frac{RI_0^2T}{2} \qquad (48)$$

Hence,

$$T = CR = \frac{C}{G} \qquad (49)$$

In circuits having resistance and condensance in series, as in Fig. 44, the time constant is equal to the condensance divided by the conductance.

Equations (47) and (49) are of fundamental importance in the study of transient phenomena. The exponential equation for the transients in Figs. 44 and 46 may be rewritten using the value of T as given in equations (47) and (49) and the data in the circuit diagrams.

Fig. 44, equations (39), (49):

$$i = I_0\epsilon^{-\frac{t}{T}} = I_0\epsilon^{-\frac{G}{C}t} = 0.127\epsilon^{-48t} \text{ amp.} \tag{50}$$

Fig. 44, equations (40), (49):

$$e = E_0\epsilon^{-\frac{t}{T}} = E_0\epsilon^{-\frac{G}{C}t} = 86\epsilon^{-48t} \text{ volts} \tag{51}$$

Fig. 45, equations (39), (47):

$$i = I_0\epsilon^{-\frac{t}{T}} = I_0\epsilon^{-\frac{R}{L}t} = 2.85\epsilon^{-44.5t} \text{ amp.} \tag{52}$$

Fig. 45, equations (40), (47):

$$e = E_0\epsilon^{-\frac{t}{T}} = E_0\epsilon^{-\frac{R}{L}t} = 36\epsilon^{-44.5t} \text{ volts} \tag{53}$$

Fig. 46, equations (44), (47):

$$i = I - I\epsilon^{-\frac{t}{T}} = I - I\epsilon^{-\frac{R}{L}t} = 4.5 - 4.5\epsilon^{-53.6t} \text{ amp.} \tag{54}$$

The reciprocals of the time constants appearing in the exponential equations as R/L and G/C, or such combinations of circuit constants as the complexity of the system may require, are often called the *dissipation constants* or the *attenuation constants* of the circuit.

The expressions for the time constants in equations (47) and (49) may be derived from the current-time and voltage-time curves instead of basing the equations directly on the principle of the conservation of energy. In Fig. 44 the quantity of electricity (coulombs) in the condenser when starting the transient must be equal to the total amount

expended when the condenser is discharged, as represented by the area between the current-time curve and the X-axis.

$$CE_0 = \int_o^\infty idt = I_0 \int_o^\infty \epsilon^{-\frac{t}{T}} dt = I_0 T \tag{55}$$

Hence,

$$T = C\frac{E_0}{I_0} = CR = \frac{C}{G} = 0.021 \text{ sec.} \tag{56}$$

Similarly, in Fig. 45, the magnetic flux in the field when starting the transient may be equated to the total number of lines of force cutting the circuit when all the magnetic energy in the field changes into heat in the resistance.

$$LI_0 = \int_o^\infty edt = R\int_o^\infty idt = RI_0 \int_o^\infty \epsilon^{-\frac{t}{T}} dt = RI_0 T \tag{57}$$

Hence,

$$T = \frac{L}{R} = 0.023 \text{ sec.} \tag{58}$$

If the initial rate of discharge in Fig. 44 be continued unchanged, the current-time curve would coincide with the line OQ and the condenser would be completely discharged in the time represented by PQ or T. Hence the area of the rectangle $OPQN$ must be equal to the area between the current-time curve and the X-axis.

Similarly, in Fig. 45, if the initial rate of discharge continued unchanged, the current-time curve would coincide with the line OQ and all the energy stored in the magnetic field would appear as heat in the resistance in the time represented by PQ or T. Hence the area of the rectangle $OPQN$ must be equal to the area between the current-time curve and the X-axis.

Expressions for the transient current and voltage as given in equations (50), (51), (52), (53) and (54) may be derived without using the time constant term. The customary differential equations giving the basic relations, with expres-

sions for the transient currents, are given in equations (59) to (62).

For circuits having resistance and condensance, Fig. 44, while the condenser is discharged through a constant resistance:

$$Ri + \int \frac{idt}{C} = 0; \text{ hence } i = I_0 \epsilon^{-\frac{t}{RC}} = I_0 \epsilon^{-\frac{G}{C}t} \qquad (59)$$

For circuits having resistance and condensance, as in Figs. 44 and 48, the transient current while charging from 0 to the voltage E is the same as in discharging through the resistance.

$$Ri + \int \frac{idt}{C} = E; \text{ hence } i = I_0 \epsilon^{-\frac{1}{RC}t} \qquad (60)$$

For circuits having resistance and inductance, Fig. 46, while the magnetic field is formed:

$$Ri + L\frac{di}{dt} = E; \text{ hence, } i = \frac{E}{R} - \frac{E}{R}\epsilon^{-\frac{R}{L}t} = I - I\epsilon^{-\frac{R}{L}t} \qquad (61)$$

For circuits having resistance and inductance, Fig. 45, and á magnetic field produced by a current I_0, while the field discharges through a constant resistance:

$$Ri + L\frac{di}{dt} = 0; \text{ hence, } i = I_0 \epsilon^{-\frac{R}{L}t} \qquad (62)$$

The Exponential Curve.—Oscillograms of simple electric transients give photographic records of the current-time factors. The amplitude of the curve varies directly with the magnitude of the current passing through the vibrator and the strength of the magnetic field in which the vibrator moves. The length of film used for any given unit of time depends on the speed of the revolving drum carrying the film. It is evident that both the amplitude of the mirror vibrations and the speed of the film may be adjusted independently of the circuit in which the transient occurs. By examining exponential equations representing simple electric transients it is apparent that if the value of the time

constant T be used as the unit of length on the X-axis and the initial value of the variable as the unit of measure for the ordinates, then all exponential transients will have the same shape and may be represented by the numerical values of the exponential equation $y = \epsilon^{-x}$. The same space unit need not be used on both axes to represent the unit values of current and time, but the scale may be selected so as to secure a convenient shape for the available space. In Fig. 49 is shown a current-time curve in which

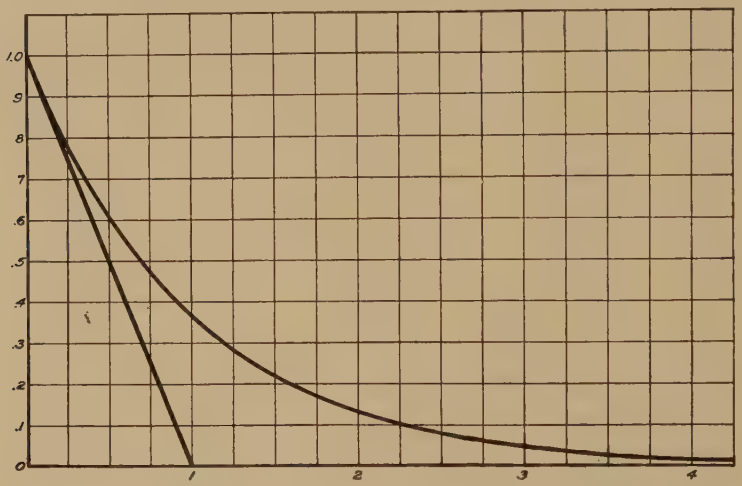

FIG. 49.—The exponential curve. Current-time transient.

the unit representing the initial value of the transient current is 5 cm., while the unit used on the X-axis, that is, for the time constant of the circuit, is 2 cm.

By using the initial value of the transient as the unit of ordinates and the time constant of the circuit as the unit of abscissæ, all exponential transients are of the same shape and if plotted to the same scale would be identical with the curve in Fig. 49. The numerical relations between y and x in the exponential equation $y = \epsilon^{-x}$ are given in Table III. While the plotting of transients may be facilitated by the selection of the above units, the actual initial values of the transient quantity, expressed in amperes or volts

may be of any magnitude as determined by the circuit conditions.

TABLE III

$y = \epsilon^{-x}$; $\epsilon = 2.71828$

x	y	x	y
0.00	1.000	1.2	0.301
0.05	0.951	1.4	0.247
0.10	0.905	1.6	0.202
0.15	0.860	1.8	0.165
0.20	0.819	2.0	0.135
0.30	0.741	2.5	0.082
0.40	0.670	3.0	0.050
0.50	0.607	3.5	0.030
0.60	0.549	4.0	0.018
0.70	0.497	4.5	0.011
0.80	0.449	5.0	0.007
0.90	0.407	6.0	0.002
1.00	0.368	7.0	0.001

Initial Transient Values.—In simple electric transients the initial value of the variable quantity depends on both the permanent value and on the relative magnitude of the circuit constants. Thus equations (47) and (58) show that the time constant of a magnetic field depends on the inductance and resistance in the circuit. If the energy stored in the magnetic field be discharged by short circuiting the terminals of the field, the initial value of the transient voltage will be equal in magnitude but opposite in direction to the previously permanent value. But if the discharge be made through an additional resistance R_2, the initial voltage transient will be greater in magnitude in the ratio of $R_1 + R_2 : R_1$, when R_1 represents the resistance of the field winding. The time constant of the circuit in which the transients appear would be

$$T = \frac{L}{R_1} \text{ when the field is short circuited,}$$

and $T_1 = \dfrac{L}{R_1 + R_2}$ when the additional resistance R_2 is

inserted in the discharging circuit. With the same amount of energy stored in the magnetic field, the products of the initial value and the corresponding time constants must be equal.

$$E_0 T = E_0' T' \tag{63}$$

Hence,

$$E_0 : E_0' :: R_1 : R_1 + R_2 \tag{64}$$

$$E_0' = E_0 \frac{R_1 + R_2}{R_1} \tag{65}$$

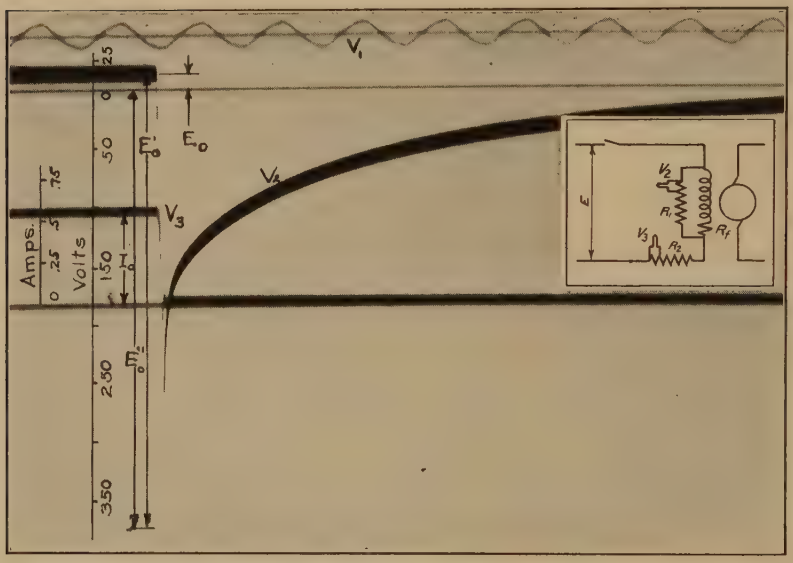

FIG. 50.—Magnetic field discharging through additional resistance R_1.
E = 13.2 volts; I_0 = 0.54 amp.; R_1 = 628.5 ohms; R_2 = 8.2 ohms; R_f = 24 ohms; $v_1 = 100\sim$.

The initial induced discharge voltage is therefore greater than the permanent impressed voltage in the proportion of the resistances in circuit for the two cases. In the voltage-time curve, Fig. 50, the initial discharge voltage E_0'' is that part of the induced voltage E_0' due to the $R_2 i$ drop.

$$E_0'' = E_0' \frac{R_2}{R_1 + R_2} = E_0 \frac{R_2}{R_1} \tag{66}$$

This relation is of great importance in the design and operation of electrical machinery. In breaking electric circuits, as induction coils, motor and generator fields, transmission lines, etc., in which energy is stored magnetically, the air or oil gap in the switch introduces a rapidly increasing resistance. The faster the contact points of the switch or circuit breaker separate, the more rapidly the resistance is inserted and the higher the induced voltage.

Figure 51 shows the voltage-time and current-time oscillograms for breaking the field circuit of a direct-current motor. In opening the switch an arc is formed by which a resistance of rapidly increasing magnitude is introduced into the circuit. The oscillogram shows that in about one-fifteenth of a second the induced voltage increased to more than 15 times the voltage impressed on the terminals of the field before the switch was opened. Although the voltage applied to the motor field was only 74.5 volts, the opening of the switch in the field circuit produced a transient stress of over 1,100 volts on the field insulation. Since the transient induced voltage on the motor field winding is directly proportional to the rate of cutting lines of force the shorter the time used in opening the switch, or the faster the resistance is inserted in the circuit, the greater the transient voltage stress tending to puncture the field insulation. If the circuit breaker operates in steps by which resistances of known value are introduced into the circuit in rapid succession, the transient induced voltage will be proportionately lower and the destructive action of the arc greatly reduced. Since the energy stored in an electromagnetic field depends on the current flowing in the field windings, it must be converted into some other form when the current is interrupted.

Current, Voltage and Magnetic-flux Transients.—In electromagnetic circuits having constant permeability the current, voltage and flux transients have the same shape

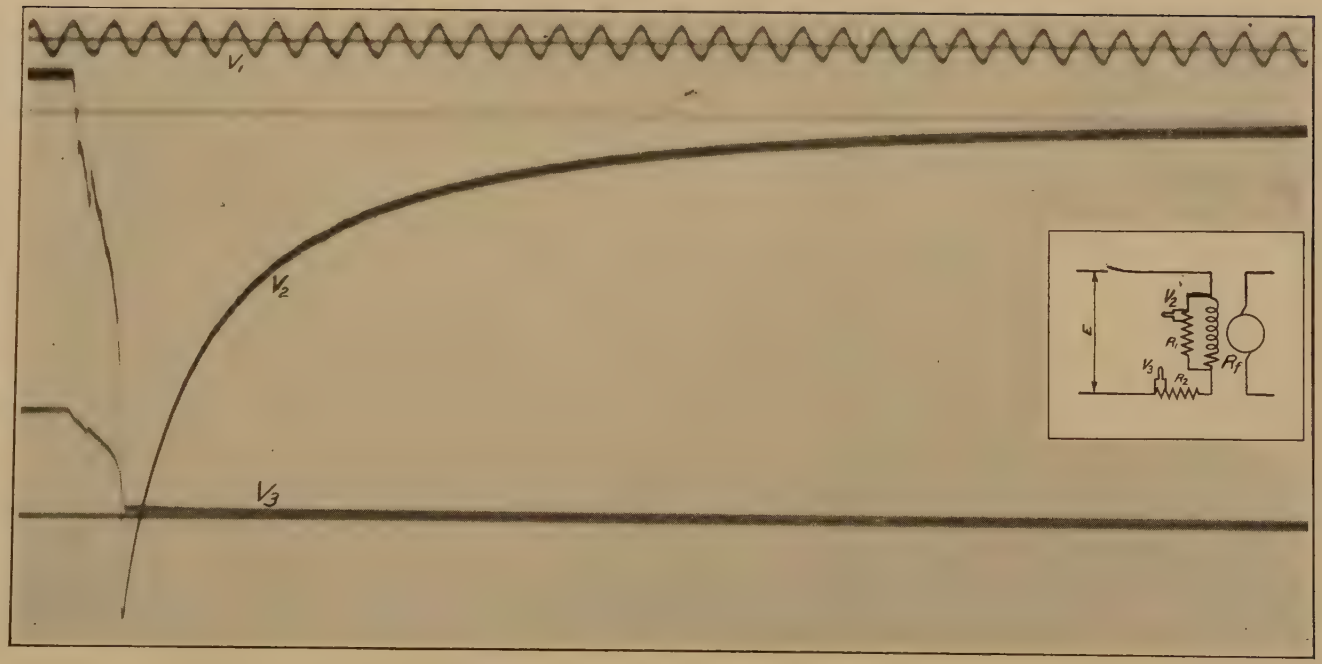

FIG. 51.—Breaking field circuit of a direct-current motor.
E = 74.5 volts; I = 1.5 amp.; R_1 = 1,090 ohms; R_f = 49.6 ohms; v_1 = 100∼.

and are expressed by the exponential equation. Referring to Fig. 45

$$e = Ri \tag{67}$$

The curve in the oscillogram therefore represents either the current or voltage transients and the quantitative values are obtained by applying the corresponding ampere and volt scales. From the law of electromagnetic induction the induced voltage is equal to the rate of cutting lines of force.

$$e = -\frac{d\phi}{dt} 10^{-8} \text{ volts} \tag{68}$$

Hence,

$$\phi = -10^{8} \int edt = \frac{EL10^{8}}{R} \epsilon^{-\frac{R}{L}t}$$

$$= (\text{constant}) \epsilon^{-\frac{R}{L}t} \text{ lines of flux} \tag{69}$$

The flux transient therefore is an exponential curve of the same form as the current and voltage transients.

In Fig. 52 is shown the corresponding transients for the current, voltage and flux in forming an electromagnetic field, as in Fig. 46, in a magnetic circuit of constant permeability. The transients are shown by the dotted lines, the permanent values by the broken lines and the instantaneous values by the full-line curves.

$$i = I + \left(-I\epsilon^{-\frac{t}{T}}\right) = I - I\epsilon^{-\frac{R}{L}t} \tag{70}$$

$$e = E + \left(-E\epsilon^{-\frac{t}{T}}\right) = E - E\epsilon^{-\frac{R}{L}t} \tag{71}$$

$$\phi = \Phi + \left(-\Phi\epsilon^{-\frac{t}{T}}\right) = \Phi - \Phi\epsilon^{-\frac{R}{L}t} \tag{72}$$

Equations (70), (71) and (72) express the instantaneous values as equal to the algebraic sum of the permanent and transient quantities. At any instant in time t, as indicated in Fig. 52:

PQ = the instantaneous value, i, e or ϕ (73)

PS = the permanent or final value, I, E or ϕ (74)

PN = the transient value, $I_0\epsilon^{-\frac{t}{T}}$, $E_0\epsilon^{-\frac{t}{T}}$, $\Phi_0\epsilon^{-\frac{t}{T}}$ (75)

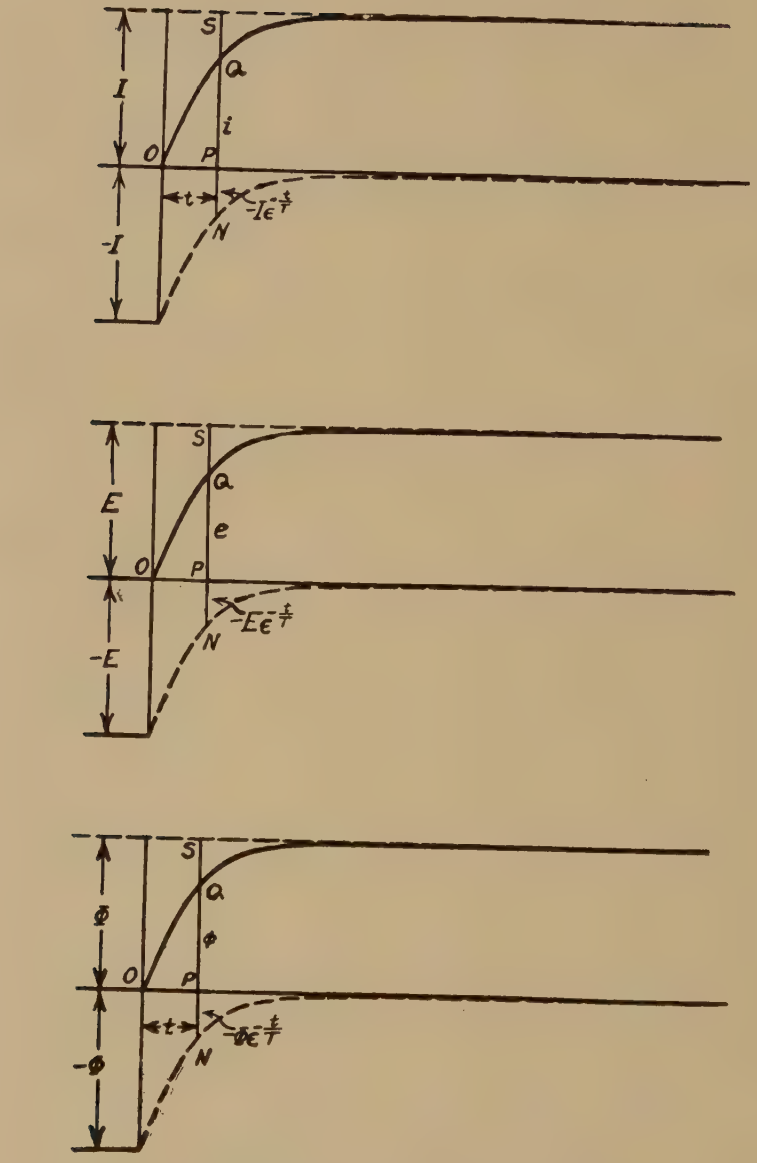

FIG. 52.—Single-energy voltage, current and magnetic-flux transients in forming a magnetic field in a magnetic circuit of constant permeability.

In discharging energy stored in a magnetic field, in a magnetic circuit of constant permeability, through a constant resistance, the transient and the instantaneous values are equal since the permanent value is zero.

$$i = I_0\epsilon^{-\frac{t}{T}} = I_0\epsilon^{-\frac{R}{L}t} \tag{76}$$

$$e = E_0\epsilon^{-\frac{t}{T}} = E_0\epsilon^{-\frac{R}{L}t} \tag{77}$$

$$\phi = \Phi_0\epsilon^{-\frac{t}{T}} = \Phi_0\epsilon^{-\frac{R}{L}t} \tag{78}$$

Problems and Experiments

1. A condenser of 115 mf., charged to 500 volts, is discharged through a constant resistance of 425 ohms.

(*a*) Derive the equations for the current-time curve.

(*b*) Find the time constant of the circuit.

(*c*) Plot the voltage-time curve across the terminals of the condenser, during discharge. Ordinates in volts and abscissæ in seconds.

(*d*) Draw an ampere scale of ordinates so that the curve plotted in (*c*) will represent the current transient.

2. The time constant of an inductance coil is found by taking an oscillogram to be 0.04 sec. The resistance in circuit was 15.8 ohms.

(*a*) Find the inductance in henrys.

(*b*) With 110 volts impressed on the coil, plot the starting current transient.

(*c*) Write the equation for the current-time curve in (*b*).

3. From the data given in Fig. 31 find the time constant of the circuit. Derive expression for i in amperes; ${}_Re$ in volts, numerical values.

4. From the data given in Fig. 48 find the time constant of the circuit. Find numerical value of i in amperes and ${}_Re$ in volts.

5. Take an oscillogram of the starting-current transient of the field of a laboratory motor or generator.

(*a*) From the oscillogram find the time constant of the field.

(*b*) Measure the resistance of the circuit and calculate the inductance of the field.

6. With the vibrators connected as shown in the circuit diagram in Fig. 51 take an oscillogram showing the current and voltage transients produced by breaking the field circuit of a motor or generator.

7. By means of oscillograms determine the time required for the operation of automatic circuit breakers. Arrange the connection for the vibrators so as to show the time consumed by each step in the operation.

CHAPTER IV

SINGLE-ENERGY TRANSIENTS. ALTERNATING CURRENTS

In direct-current systems the transient electric phenomena described in the preceding chapter are due to the storage of energy in magnetic and dielectric fields. If a constant direct-current voltage is impressed on a circuit having constant resistance but neither inductance nor condensance, the current would instantly reach its permanent value, and any change in the voltage would at the same time cause a proportional change in the current. With either condensance or inductance in the circuit, a short period of time is required for the current to reach its permanent value after any change in voltage, and the current-time curve during the transition period is expressed by the exponential equation. The value of the variable current is at any instant equal to the algebraic sum of the permanent and transient values; and single-energy transients for direct currents in circuits having constant resistance and inductance, or constant resistance and condensance, may be expressed by exponential equations similar in form to equations (70), (71) and (72).

Single-phase, Single-energy, Load Circuit Transients.—The same principle applies to single-energy transients in alternating-current systems. In circuits having constant resistance but neither inductance nor condensance, no transients appear. Any change in the voltage produces instantly a proportionate change in the current. In circuits having either inductance or condensance a transient period for the readjustment of the energy content of the system follows any change in voltage. At any instant the actual current or voltage is the algebraic sum of the

corresponding permanent and transient values. The permanent term i' is the alternating-current wave assumed to be of simple sine form as expressed in equation (82) with γ_1 as the time phase angle at the starting moment.

$$i' = {}^{M}I \sin (\omega t - \gamma_1) \tag{82}$$

$$e' = {}^{M}E \sin (\omega t - \gamma_2) \tag{83}$$

The transient term i'' is expressed by the exponential equation of the same form as in direct currents with an

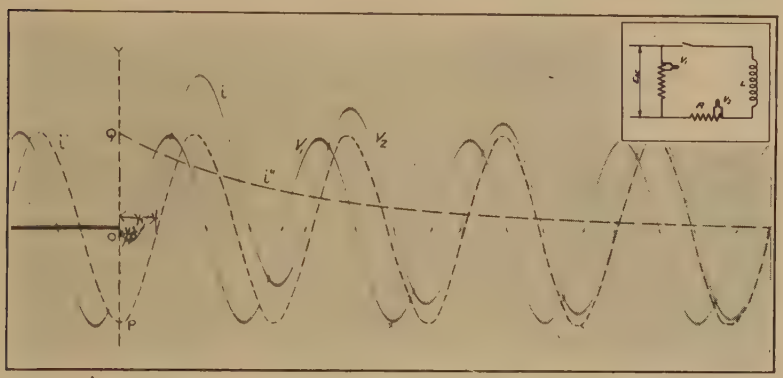

FIG. 53.—Single-phase alternator, armature current transient.
E = 100 volts; I = 1.3 amp.; R = 8.03 ohms; L = 0.201 henry; f = 60∼.

initial value equal in magnitude but opposite in time phase to what would have been the permanent value at the starting point if the circuit had been closed at some previous time.

$$i'' = {}^{M}I \sin \gamma_1 \epsilon^{-\frac{t}{T}} \tag{83a}$$

$$e'' = {}^{M}E \sin \gamma_2 \epsilon^{-\frac{t}{T}} \tag{84}$$

The instantaneous value of the current or voltage would therefore be expressed by equations (85) and (86):

$$i = i' + i'' = {}^{M}I \sin (\omega t - \gamma_1) + {}^{M}I \sin \gamma_1 \epsilon^{-\frac{t}{T}} \tag{85}$$

$$e = e' + e'' = {}^{M}E \sin (\omega t - \gamma_2) + {}^{M}E \sin \gamma_2 \epsilon^{-\frac{t}{T}} \tag{86}$$

The oscillogram in Fig. 53 shows the current-time curve i produced in a circuit having 8.03 ohms resistance and

0.201 henry inductance when a 60-cycle alternating-current voltage e was impressed by closing a switch at the instant in time represented by the OY-axis. In Fig. 54 is shown an oscillogram taken under the same circuit conditions as Fig. 53, but the switch was closed at another point on the current wave.

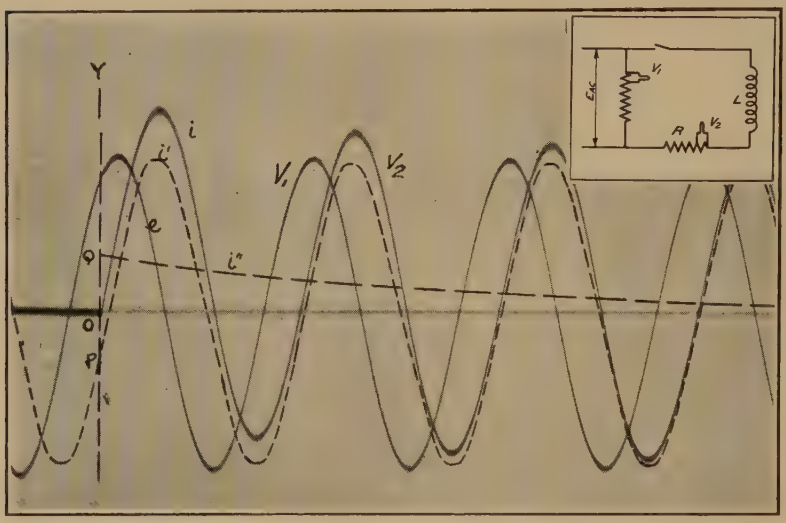

FIG. 54.—Single-phase, single-energy current transient.
$E = 100$ volts; $I = 1.3$ amp.; $R = 8.03$ ohms; $L = 0.201$ henry; $f = 60\sim$.

The voltage impressed is represented by the sine wave, Fig. 53,

$$e' = {}^{M}E \sin (\omega t - \gamma_2) \tag{87}$$

The actual current flowing is represented by the curve i, which practically coincides, after completing four cycles, with the permanent value i', shown by the dotted sine wave. The transient i'' is shown as the broken line whose initial value

$$OQ = (-OP) = {}^{M}I \sin \gamma_1 \tag{88}$$

At any instant t after the closing of the switch i is equal to the algebraic sum of i' and i''.

$$i = i' + i'' = {}^{M}I \sin (\omega t - \gamma_1) + {}^{M}I \sin \gamma_1 \, \epsilon^{-\frac{R}{L}t} \tag{89}$$

For the data given in Fig. 53:

$$i = 1.3 \sin\left(\omega t - \frac{\pi}{2}\right) + 1.3\,\epsilon^{-40t} \text{ amp.} \qquad (90)$$

It is evident that the initial value of the transient may vary from $-{}^{M}I$ to $+{}^{M}I$, depending at what point of the voltage-time curve the circuit is closed. If the switch be

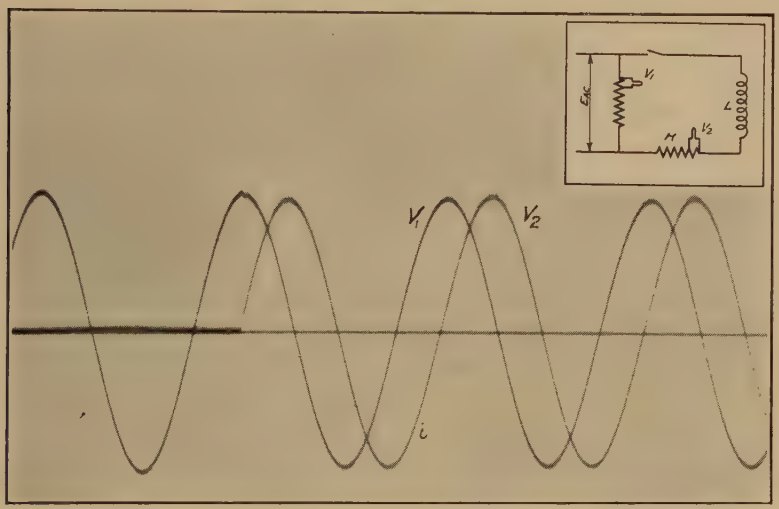

Fig. 55.—Single-phase, single-energy, showing zero current transient.
$E = 100$ volts; $I = 1.3$ amp.; $R = 8.03$ ohms; $L = 0.201$ henry; $y_1 = 0$; $f = 60\sim$

thrown at the instants when the permanent current wave would be zero, $\gamma_1 = 0$ or 180°, no transient would appear and the permanent and actual current-time curves would coincide throughout as shown in Fig. 55.

$$i = i' = {}^{M}I \sin(\omega t) \qquad (91)$$

The transient current would have a maximum initial value if the circuit is closed at the instant the permanent current wave is at a maximum, that is, when $\gamma_1 = 90°$, as in Fig. 53. The time constant for the current transient would be the same at whatever point in the cycle the

circuit is closed, as it depends on the resistance and inductance in the load circuit and not on the magnitude of the initial value of the transient current.

Three-phase, Single-energy, Load Circuit Transients.—For three-phase circuits similar relations exist. The starting current transients in three-phase systems in which energy may be stored either magnetically or dielectrically follow the same laws as discussed for single-phase circuits.

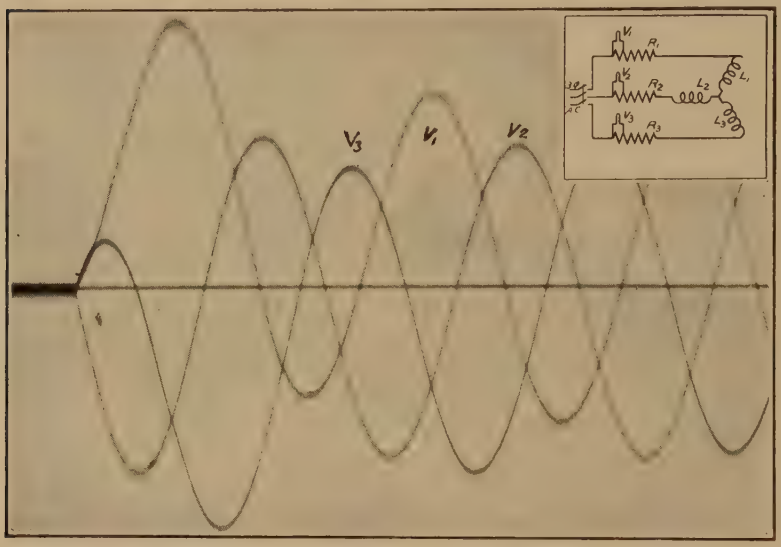

FIG. 56.—Three-phase, single-energy starting current transients. E = 120 volts; I = 2.25 amp.; $R_1 = R_2 = R_3$ = 4.43 ohms; $L_1 = L_2 = L_3$ = 0.0734 henry; f = 60∼.

In Figs. 57 and 58 are shown oscillograms of the three starting load currents in a three-phase system, star connected, having 10.06 ohms resistance and 0.126 henry inductance in each phase to neutral. The corresponding permanent current waves and transient currents were traced on the oscillogram in Fig. 57. In Fig. 58 the circuits were closed at the instant the current in v_1 was of zero value. The following equations give the instantaneous values of the armature currents in the three phases, Fig. 57.

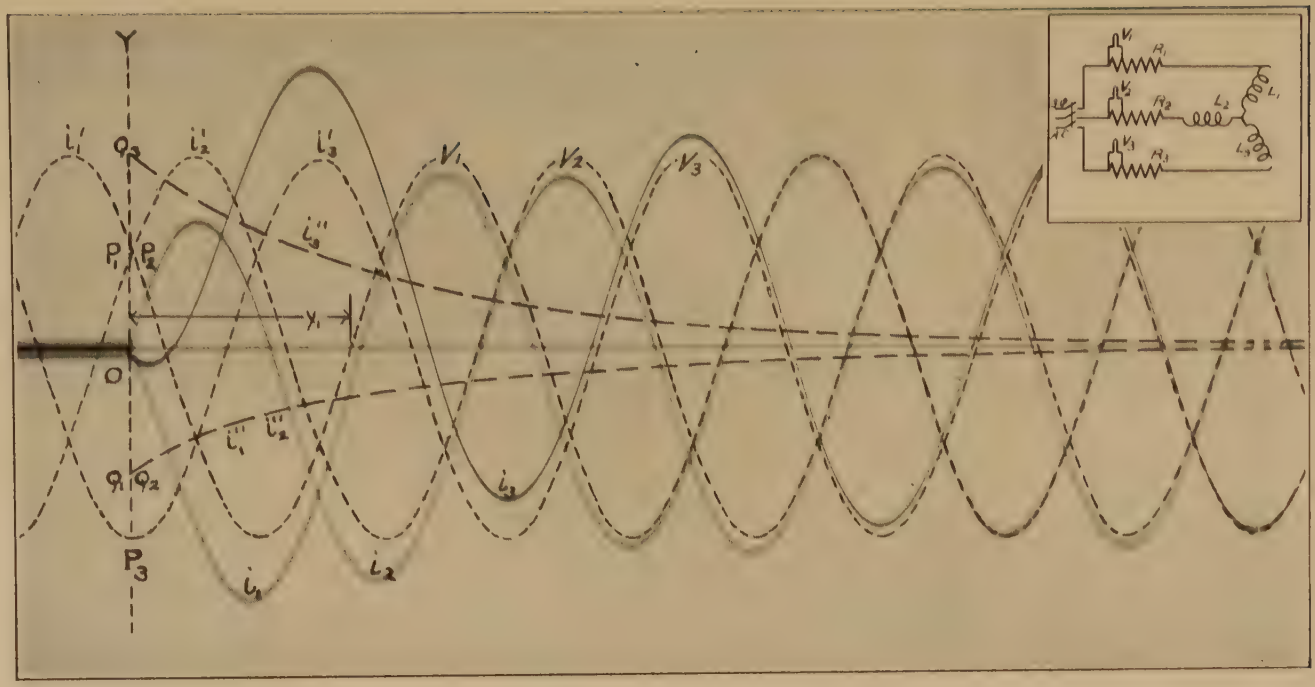

FIG. 57.—Three-phase, single-energy starting current transients.

$E = 163$ volts; $I = 1.82$ amp.; $R_1 = R_2 = R_3 = 10.06$ ohms; $L_1 = L_2 = L_3 = 0.126$ henry; $f = 60\sim$.

Phase 1:

Impressed voltage,

$$e_1 = {}^{\varkappa}E_1 \sin (\omega t - \gamma_2) \tag{92}$$

Permanent current,

$$i_1' = {}^{\varkappa}I_1 \sin (\omega t - \gamma_1) \tag{93}$$

Initial value starting current transient,

$$OQ_1 = (-OP_1) = {}^{\varkappa}I_1 \sin \gamma_1 \tag{94}$$

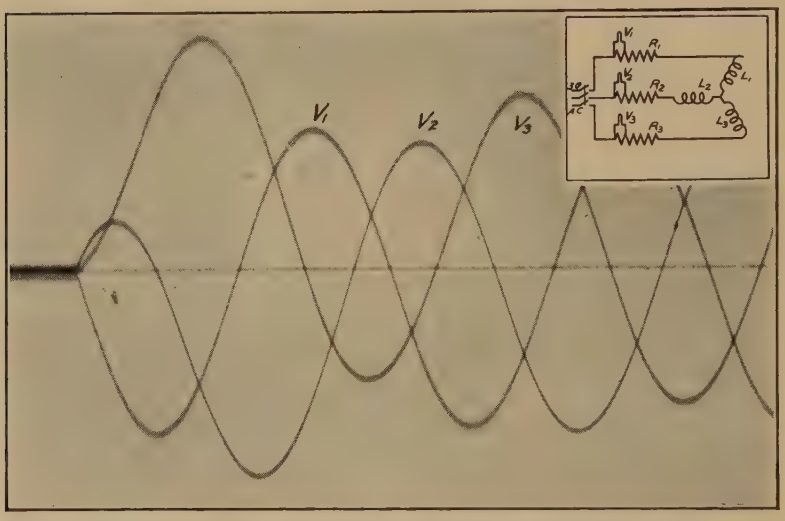

FIG. 58.—Three-phase, single-energy starting current transients. $E = 163$ volts; $I = 1.82$ amp.; $R_1 = R_2 = R_3 = 10.06$ ohms; $L_1 = L_2 = L_3 = 0.126$ henry; $f = 60\sim$.

Transient current,

$$i_1'' = {}^{\varkappa}I_1 \sin \gamma_1 \, \epsilon^{-\frac{R_1}{L_1}t} \tag{95}$$

Actual current, oscillogram,

$$i_1 = i_1' + i'' = {}^{\varkappa}I_1 \sin (\omega t - \gamma_1) + {}^{\varkappa}I_1 \sin \gamma_1 \epsilon^{-\frac{R_1}{L_1}t} \tag{96}$$

Time constant,

$$T = \frac{L_1}{R_1} = 0.013 \text{ sec.} \tag{97}$$

Phase 2:

Impressed voltage,

$$e_2 = {}^{M}E_2 \sin(\omega t - \gamma_2 - 120°) \tag{98}$$

Permanent current,

$$i_2' = {}^{M}I_2 \sin(\omega t - \gamma_1 - 120°) \tag{99}$$

Initial value starting current transient,

$$OQ_2 = (-OP_2) = {}^{M}I_2 \sin(\gamma_1 + 120°) \tag{100}$$

Transient current,

$$i_2'' = {}^{M}I_2 \sin(\gamma_1 + 120°)\epsilon^{-\frac{R_2}{L_2}t} \tag{101}$$

Actual current, oscillogram,

$$i_2 = i_2' + i_2'' = {}^{M}I_2 \sin(\omega t - \gamma_1 - 120°) + {}^{M}I_2 \sin(\gamma_1 + 120°)\epsilon^{-\frac{R_2}{L_2}t} \tag{102}$$

Time constant,

$$T_2 = \frac{L_2}{R_2} = 0.013 \text{ sec.} \tag{103}$$

Phase 3:

Impressed voltage,

$$e_3 = {}^{M}E_3 \sin(\omega t - \gamma_2 - 240°) \tag{104}$$

Permanent current,

$$i_3' = {}^{M}I_3 \sin(\omega t - \gamma_1 - 240°) \tag{105}$$

Initial value starting current transient,

$$OQ_3 = (-OP_3) = {}^{M}I_3 \sin(\gamma_1 + 240°) \tag{106}$$

Transient current,

$$i_3'' = {}^{M}I_3 \sin(\gamma_1 + 240°)\epsilon^{-\frac{R_3}{L_3}t} \tag{107}$$

Actual current, oscillogram,

$$i_3 = i_3' + i_3'' = {}^{M}I_3 \sin(\omega t - \gamma_1 - 240°) + {}^{M}I_3 \sin(\gamma_1 + 240°)\epsilon^{-\frac{R_3}{L_3}t} \tag{108}$$

Time constant,

$$T_3 = \frac{L_3}{R_3} = 0.013 \text{ sec.} \tag{109}$$

It is of interest to note that the sum of the instantaneous values of the three currents, $i_1 + i_2 + i_3$, is equal to zero during the starting period as well as after the permanent state has been reached. This is evidently the case since under permanent conditions the sum of the currents is at any instant equal to zero and hence at the instant the circuit is closed $OP_1 + OP_2 + OP_3 = 0$. Therefore, the sum of the initial values of the transient currents, $OQ_1 + OQ_2 + OQ_3 = 0$, and as the time constants of the three transients are equal the sum of the actual currents in the three phases must at any instant be equal to zero.

Starting Transient of a Polyphase Rotating Magnetic Field.—In the preceding illustrations the single-energy transients are due to changes in the amounts of energy stored in the given circuits, and the current-time curves show a continuous decrease of the current as expressed by the exponential equation. If the permanent condition relates to interconnected circuits which permit a transfer of energy from one circuit to another, although the total amount of energy stored in the magnetic field is constant, as in the rotating field of a polyphase induction motor, pulsations will appear during the transition period, which merit attention.

In Fig. 59 let a vector of constant length ON, rotating in a counter-clockwise direction represent a constant rotating magnetic field, as would be produced by three equal magnetizing coils, placed 120° apart, and excited by three-phase currents, as, for example, in a three-phase induction motor. For simplicity let the rotor be removed and consider the stator circuit and the magnetic flux during the starting transition period in which the rotating field is built up to its constant permanent value. Let the switch, connecting the stator circuit to three-phase mains, be closed at the instant the rotating magnetic flux vector ON lies along the X-axis, ON_0 in Fig. 59; as would have been the case if the switch had been closed at some previous time. The actual value of the rotating flux at the instant the

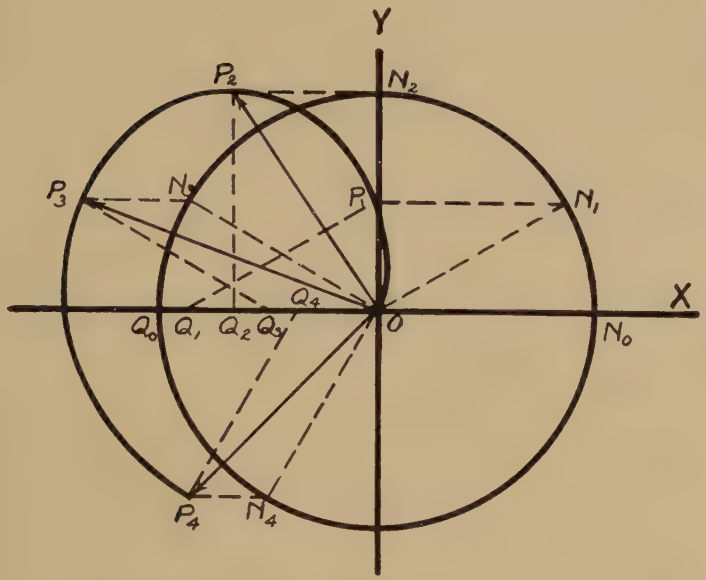

FIG. 59.—Permanent, transient and instantaneous values of the magnetic flux in starting a rotating magnetic field. Polar coördinates.

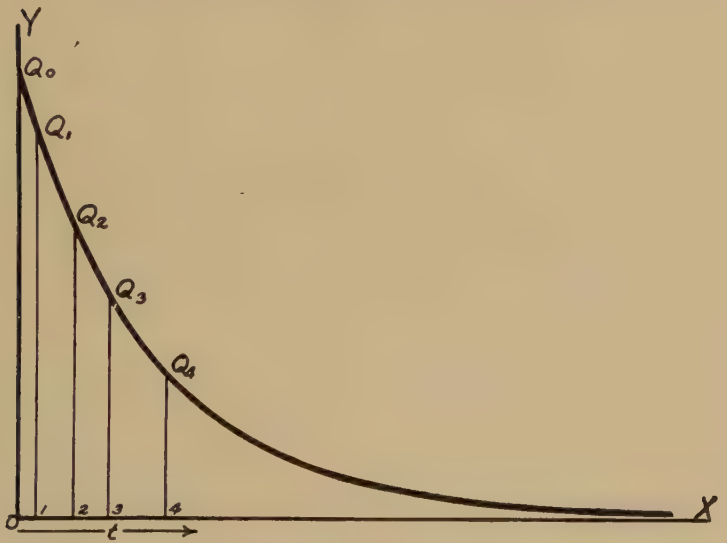

FIG. 60.—Starting magnetic flux transient from Fig. 59. Rectangular coördinates.

circuit is closed is zero. The permanent value is represented by ON_0 and since the initial value of the transient flux must be equal in magnitude but of opposite time phase, it is represented by the vector OQ_0.

$$OQ_0 = (-ON_0) \tag{110}$$

From its initial values OQ_0 the transient flux decreases in magnitude, as indicated by the exponential flux-time curve in Fig. 60, but continues fixed in space direction along the X-axis. After the time t_1 has elapsed, represented by the time angle N_0ON_1, the transient has a value OQ_1. The actual value of the flux must be the vector sum of the permanent value ON_1 and the transient OQ_1, or the resultant OP_1. In the time t_2, the transient has decreased to OQ_2 and the permanent flux vector reached the position ON_2. The actual flux OP_2 is the resultant of OQ_2 and ON_2. Similarly, OP_3 is the resultant of OQ_3 and ON_3; OP_4, of OQ_4 and ON_4, etc. From the vector diagrams, Figs. 59 and 61, it is evident that the actual starting flux will oscillate, having values greater and smaller than the permanent value, the number of oscillations depending on the time constant of the circuit. The maximum value of the flux in the starting period will in any case be less than double the permanent value, as the transient flux continuously decreases from an initial value equal to the permanent flux in magnitude and different by 180° in time phase.

The flux-time curve in Fig. 62 gives in rectangular coördinates the same relation as shown by the flux vector OP in the polar diagram in Fig. 61.

Polyphase Short Circuits. Alternator Armature and Field Transients.—Consider a three-phase alternator carrying a constant balanced load of constant power factor. The three-phase currents flowing in the armature produce a resultant constant armature flux or armature reaction. With respect to the field the armature flux is stationary, but with respect to any diameter of the armature taken as a reference axis the armature currents produce a constant

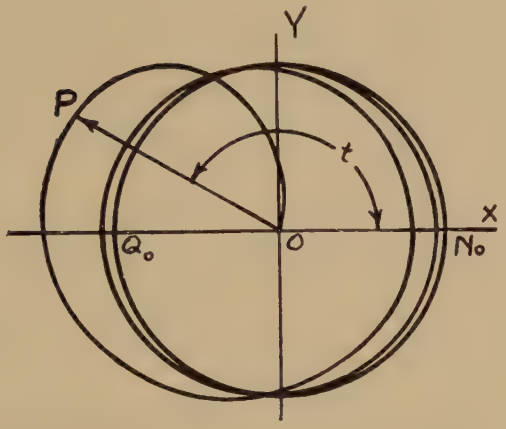

FIG. 61.—Starting a rotating magnetic field. Polar vector diagram of magnetic flux for three cycles of Fig. 59.

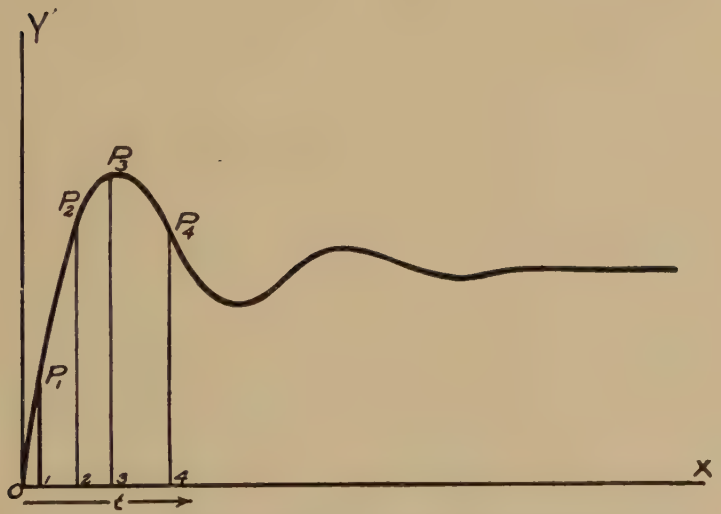

FIG. 62.—Same data as in Figs. 59 and 61. Rectangular coördinates.

rotating field of the same nature as the constant rotating field of a three-phase induction motor. For a machine in which the field is on the rotating spider while the armature is stationary, the resultant flux producing armature reaction rotates synchronously with the field. For an alternator with the field stationary and the armature rotating, the resultant constant flux rotates at the same speed but in direction opposite to the rotation of the armature and therefore is stationary with respect to the frame of the machine. In either case the armature flux or the armature reaction is stationary, if referred to the alternator field, but is a synchronously rotating field with respect to the armature.

Due to the close proximity of the armature conductors to the field poles a large part of the magnetic flux produced by the armature currents passes through the field magnetic circuit. This causes a reduction in the field flux and therefore in the amount of energy stored magnetically by the exciting current in the field. Hence, although a constant direct-current voltage is impressed on the field circuit, the useful flux is greatly reduced by the armature reaction, and as a consequence the generated armature voltage decreases in the same proportion. To effect any change in the amount of energy stored magnetically takes time, and therefore the interaction of the armature flux with the field magnetic circuit produces electric transients.

During the transition period following the instant the short circuit occurs, two distinct causes are therefore superimposed in producing transient phenomena in the interlinked electric and magnetic circuits of polyphase alternators.

(*a*) The armature transient which is equivalent to the starting transient of a rotating magnetic field having full frequency pulsations as illustrated in Figs. 61, 62.

(*b*) A field transient due to the reduction of the field flux by the armature reaction.

The transients produced under (*a*) and (*b*) differ in duration, the ratio being in each case determined by the

relative time constants of the armature and field circuits. In general, the time constant in the field circuit is greater than in the armature circuits. Large turbo-alternators have very slow field transients as compared to the duration of the armature transients.

Figures 63 and 64 show oscillograms of transients produced by short circuits on all three phases of a 7.5-kw.,

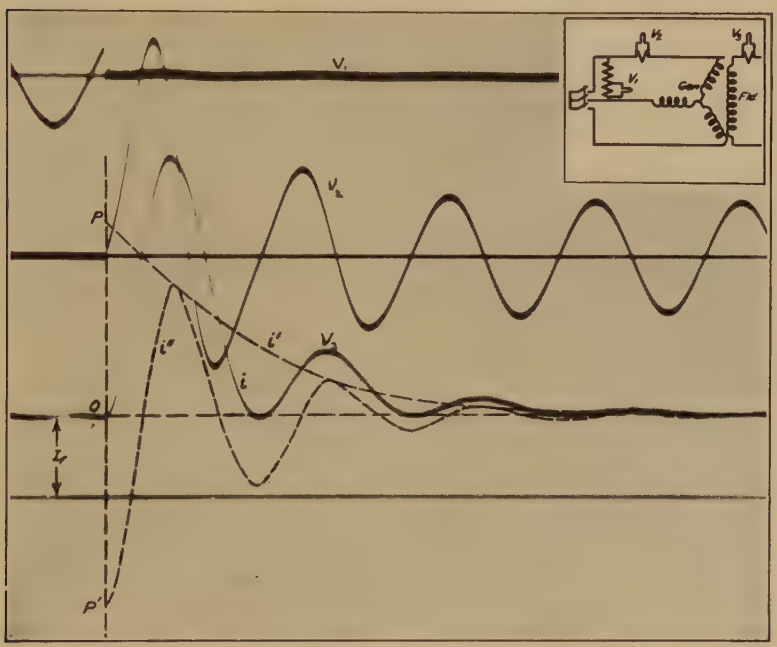

FIG. 63.—Short circuit on a three-phase, star-connected alternator. No load. E = 112 volts; I_a = 11.8 amp., short circuit; I_f = 1.33 amp.; f = 60~.

220-volt, 60-cycle, three-phase, star-connected alternator running idle and with 40 per cent normal field excitation. Similar oscillograms of short-circuit transients for the same machine while carrying 50 per cent of full load are shown in Figs. 65 and 66.

As indicated in the circuit diagrams, Figs. 63, 64, 65 and 66, vibrator v_1 records the armature voltage across one pair of slip rings e_a; vibrator v_2 the current i_a in one armature

circuit, and vibrator v_3 the field current i_f. As the short circuit is directly across v_1, the voltage e_a instantly drops to zero. The transient in the field winding is due to the combined action of the starting transient of the rotating field in the armature, which produces the full frequency pulsations, and the slower field transient resulting from the reduction of the field flux by the armature reaction. It should be noted that the field transient is of the same shape

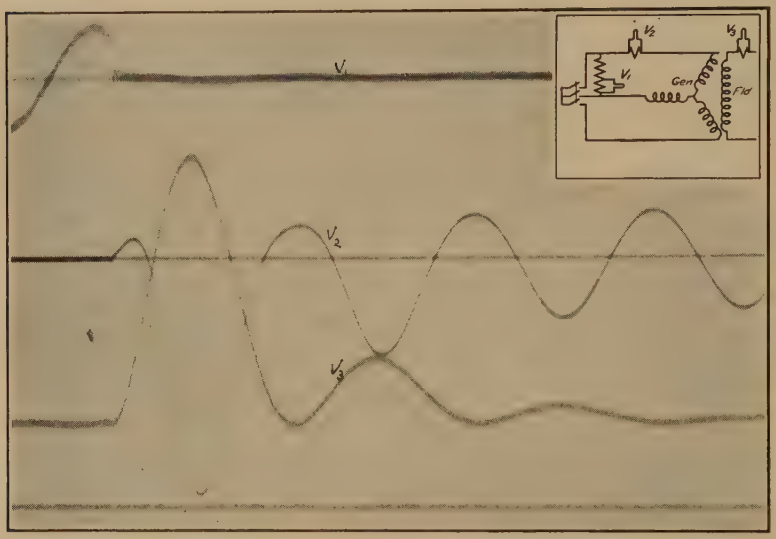

FIG. 64.—Short circuit on a three-phase, star-connected alternator. No load. $E = 112$ volts; $I_a = 11.8$ amp.; $I_f = 1.33$ amp.; $f = 60\sim$.

in the four cases. This necessarily must be the case, since the rotating field produced by balanced three-phase currents is constant in magnitude and hence the starting transient must be independent of the position of the armature conductor at the instant short circuit occurs. In breaking the short circuit the field transient alone will appear in the field winding, as shown by the oscillograms in Figs. 67 and 68. Necessarily, the transient is reversed in direction when the short circuit is broken from what is represented in Figs. 63 64, 65 and 66. It should be noted

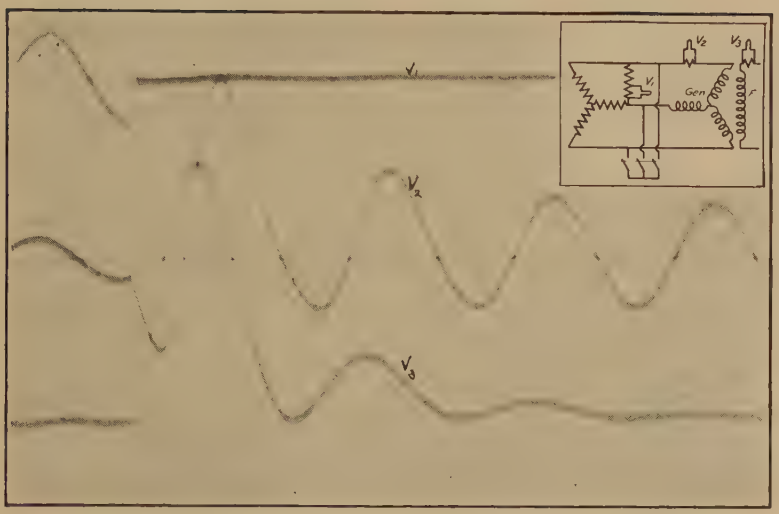

FIG. 65.—Short circuit on a three-phase, star-connected alternator. Load. $E = 108$ volts; $I_a = 5.4$ amp. with load and 12.6 amp. under short circuit; $I_f = 1.43$ amp.; $f = 60\sim$.

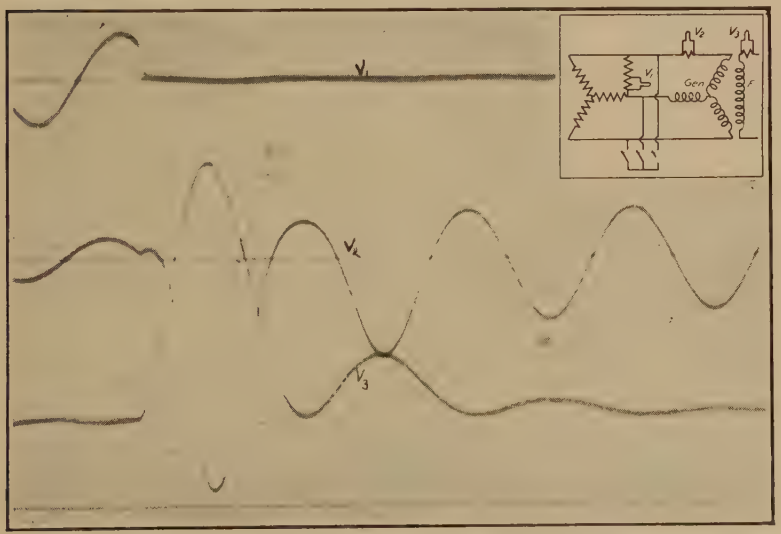

FIG. 66.—Short circuit on three-phase, star-connected alternator. Load. $E = 108$ volts; $I_a = 5.4$ amp. with load and 12.6 amp. under short circuit; $I_f = 1.43$ amp.; $f = 60\sim$.

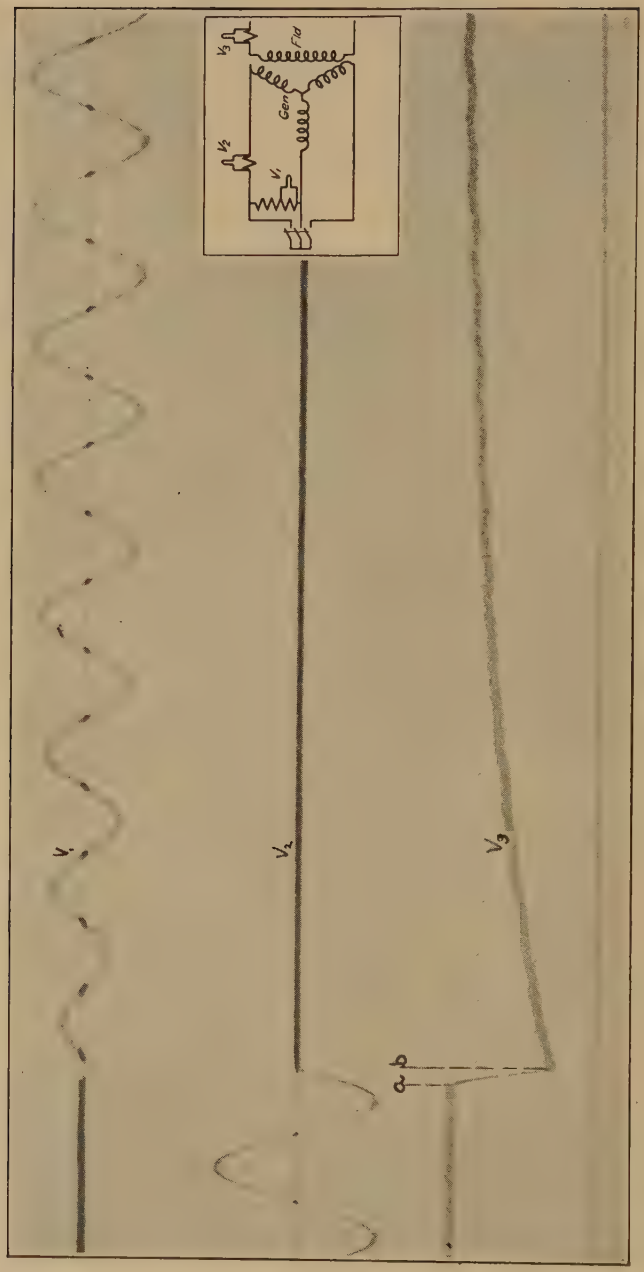

FIG. 67.—Breaking short circuit on three-phase, star-connected alternator. No load.
E = 114 volts; I_a = 13.6 amp.; I_f = 1.48 amp.; f = 60~.

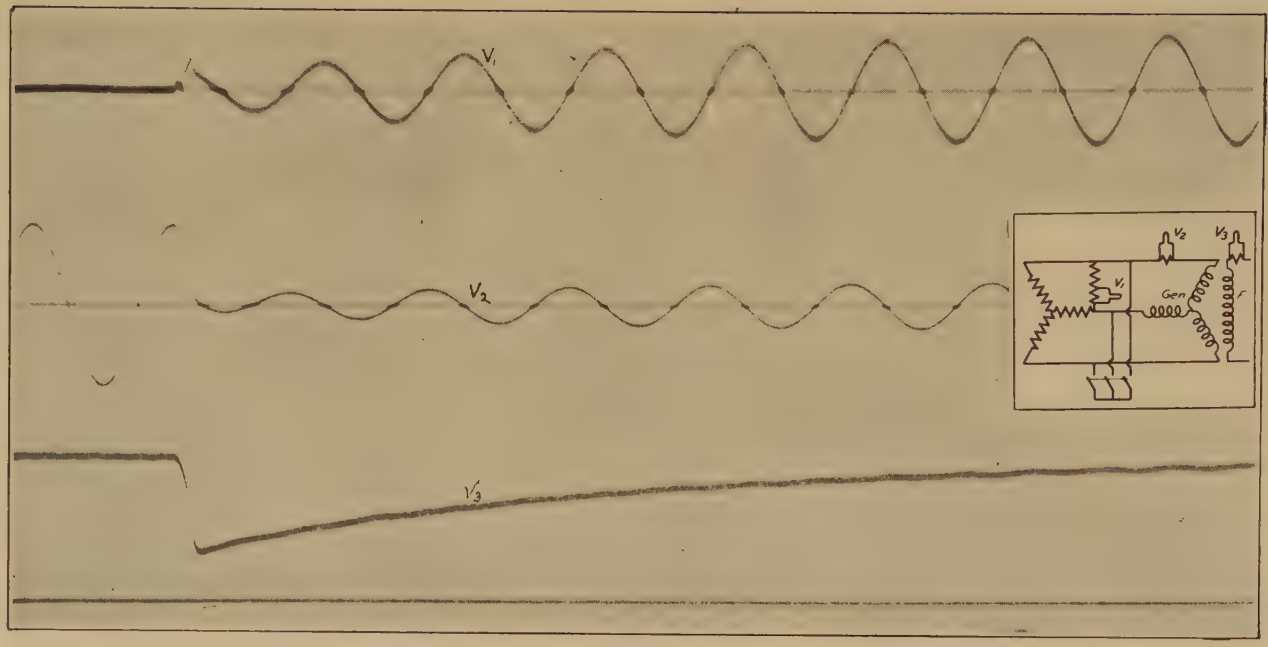

FIG. 68.—Breaking short circuit on a three-phase, star-connected alternator. Load. E = 113 volts; I_a = 5.9 amp. with load and 13.7 amp. under short circuit; I_f = 1.48 amp.; f = 60~.

that the breaking of the armature short circuit was not instantaneous, as arcs formed at the switch and continued the circuit during the time $a - b$, Fig. 67, approximately for $\frac{1}{8}$ cycle or $\frac{1}{480}$ sec. During this period the energy stored magnetically in the armature circuits was dissipated. Much more time, over 10 complete cycles, was required to restore full excitation in the field poles.

The direct relation of the voltage generated in the armature to the variable useful field flux is shown by the voltage wave e_a and the field transient i_f, Figs. 67, 68. When the short circuit is made the same transients occur, but reversed in direction, as is evident from oscillograms in Figs. 63, 64, 65 and 66.

In the operation of alternators the relative value of the initial or momentary to the final or permanent short-circuit currents is of great importance. At any instant the short-circuit current obeys Ohm's law, that is, in magnitude it will be directly as the voltage generated and inversely as the impedance of the armature circuit.

$$i_a = \frac{e_a}{z_a} = \frac{e_a}{R_a + j_r x_a} \tag{111}$$

Since the armature resistance R_a is small compared to the armature reactance ${}_r x_a$, equation (111) may be written as in equation (112).

$$i_a = \frac{e_a}{j_r x_a} \tag{112}$$

For constant speed the generated voltage e_a is directly proportional to the useful flux. At the instant the short circuit occurs and the alternator carries no load, as in Fig. 63, the useful flux depends on the direct-current voltage impressed on the field winding and produces an armature voltage ${}_o e_a$, and hence the initial or momentary value of the short-circuit current,

$${}_o i_a = \frac{{}_o e_a}{j_r x_a} \tag{113}$$

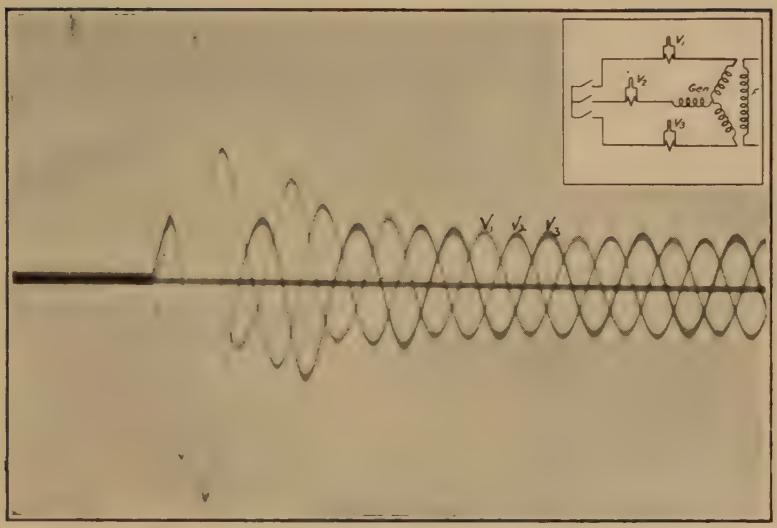

FIG. 69.—Three-phase, short-circuit armature current transients. No load. $E = 142$ volts; $I_a = 14.6$ amp.; $I_f = 1.66$ amp.; $f = 60\sim$.

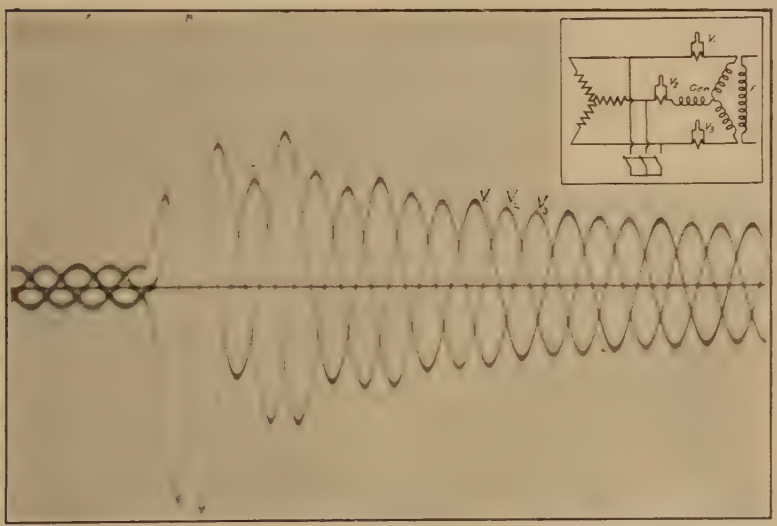

FIG. 70.—Three-phase, short-circuit armature current transients. Load. $E = 131$ volts; $I_a = 15.4$ amp., short circuit and 6.1 amp., load; $I_f = 1.75$ amp.; $f = 60\sim$.

If expressed in effective values as if the current sine wave continued at the initial magnitude,

$$ {}_{o}I_{a} = \frac{{}_{o}E_{a}}{{}_{L}x_{a}} \tag{114} $$

During the transient period following the short circuit the armature reaction reduces the field flux and as a consequence the voltage generated in the armature decreases in the same ratio. With the expiration of the field transient the useful flux ϕ_u is constant and hence the generated voltage E_a and the armature current I_a are constant or have permanent values.

$$ I_{a} = \frac{E_{a}}{{}_{L}x_{a}} \tag{115} $$

$$ \frac{I_{a}}{{}_{o}I_{a}} = \frac{E_{a}}{{}_{o}E_{a}} = \frac{\Phi_{u}}{{}_{o}\Phi_{u}} = \frac{\text{field excitation—armature reaction}}{\text{field excitation}} \tag{116} $$

Although the decrease in the armature current from its initial to the permanent value, as shown in Figs. 69 and 70, is due to a reduction in the useful field flux and hence in the generated armature voltage, it is customary to consider the voltage constant and ascribe the change to a fictitious increase in the reactance of the armature circuit. The combined effect of the armature reaction and the true armature reactance is represented by the so-called synchronous reactance ${}_{s}x_{a}$.

The permanent short-circuit current may therefore be expressed by equation (117) and the ratio of the permanent to the initial or momentary values by equation (118):

Let I_a = permanent short-circuit armature current.
${}_{o}I_{a}$ = initial short-circuit armature current.
${}_{L}x_{a}$ = armature reactance.
${}_{s}x_{a}$ = synchronous reactance = armature reactance + armature reaction.

$$ I_{a} = \frac{{}_{o}E_{a}}{{}_{s}x_{a}} \tag{117} $$

$$ \frac{I_{a}}{{}_{o}I_{a}} = \frac{{}_{L}x_{a}}{{}_{s}x_{a}} \tag{118} $$

If it be assumed that the permeability of the magnetic circuits remains constant for the changes in flux density, and that the speed of armature rotation does not change, the field current-time curve may also be expressed in the form of an equation in terms of the circuit constants and the initial value of the transients.

Let R_f = resistance of field circuit.
L_f = inductance of field circuit.
R_a = resistance of armature circuit.
L_a = inductance of armature circuit.
t = time from the instant short circuit occurs.
$\omega = 2\pi f$; f = frequency in cycles per second.
i_f = instantaneous value of field current.
I_f = permanent value of field exciting current before short circuit occurs.
i'_f = instantaneous value of current in field circuit due to field transients.
I'_f = initial value of i'_f. Due to the lag, θ_a, of the armature current and the permeability factor in the field iron.
i'_{af} = instantaneous value of current in field circuits due to armature transient.
γ_2 = phase angle of i'_{af}.
I'_{af} = initial value of i'_{af}.

$$i'_f = I'_f \epsilon^{-\frac{R_f}{L_f} t} \tag{119}$$

$$i'_{af} = I'_{af} \epsilon^{-\frac{R_a}{L_a} t} \sin(\omega t - \gamma_2) \tag{120}$$

In Figs. 67, 68:

$$i_f = I_f - i'_f = I_f - I'_f \epsilon^{-\frac{R_f}{L_f} t} \tag{121}$$

In Figs. 63, 64:

$$\begin{aligned} i_f &= I_f + i'_f + i'_{af} \\ &= I_f + I'_f \epsilon^{-\frac{R_f}{L_f} t} + I'_{af} \epsilon^{-\frac{R_a}{L_a} t} \sin(\omega t - \gamma_2) \end{aligned} \tag{122}$$

Short-circuit currents, particularly under normal field excitation, produce so great changes in flux density that the permeability is not constant and hence L_a and L_f are not constant. Short circuits also tend to change the

speed of armature rotation and hence produce variations in the frequency of the armature currents. The purpose of the equation is, however, merely to state in concise form the factors involved without taking into consideration the complications due to variations in the permeability of the magnetic circuits.

While short circuits produce electric transients of greater magnitude than the changes that occur during normal operation of alternators, it should be kept in mind that any

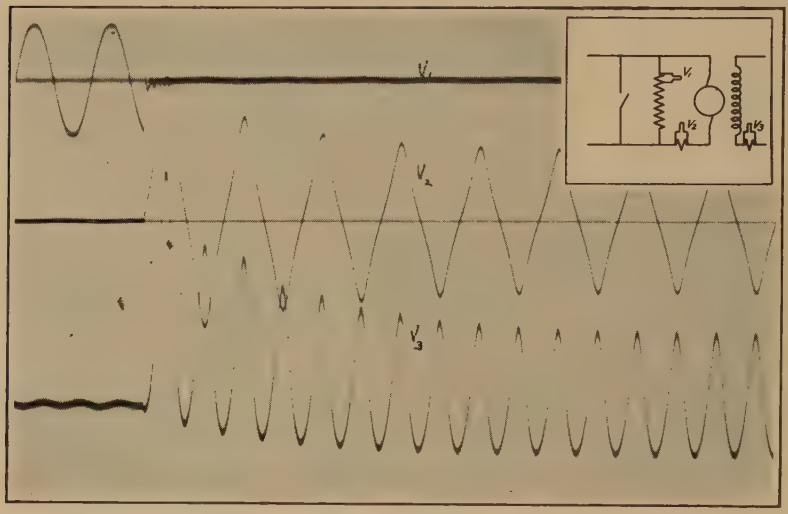

FIG. 71.—Short circuit on a single-phase alternator. No load. $E = 126$ volts; $I_a = 12.0$ amp.; $I_f = 1.29$ amp.; $f = 60\sim$.

modification in the armature currents, as, for example, an increase or a decrease in the load, produces transients having the same characteristics as those produced by short circuits. Any change in the amount of energy stored magnetically in the armature or field circuits requires time, and during the period of readjustment electric transients are produced in the interlinked electric and magnetic circuits.

Single-phase Short Circuits. Alternator Armature and Field Transients.—In polyphase alternators the permanent

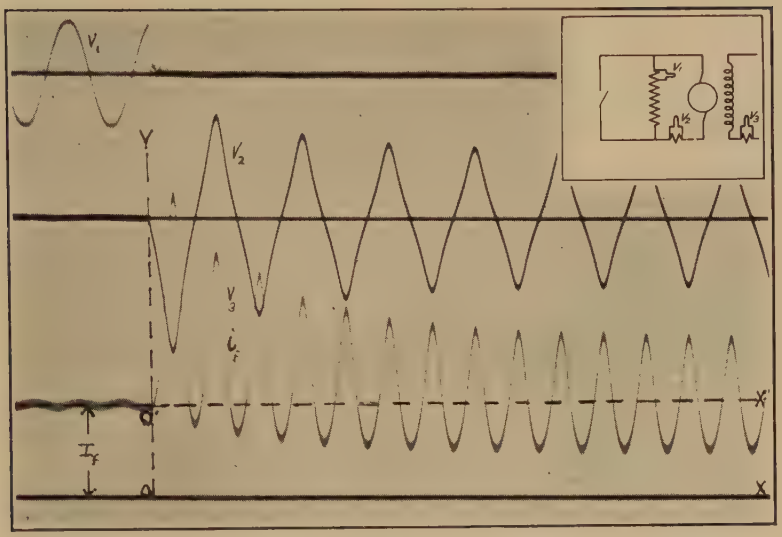

FIG. 72.—Short circuit on a single-phase alternator. No load.
$E = 126$ volts; $I_a = 12.0$ amp., short circuit; $I_f = 1.29$ amp.; $f = 60\sim$.

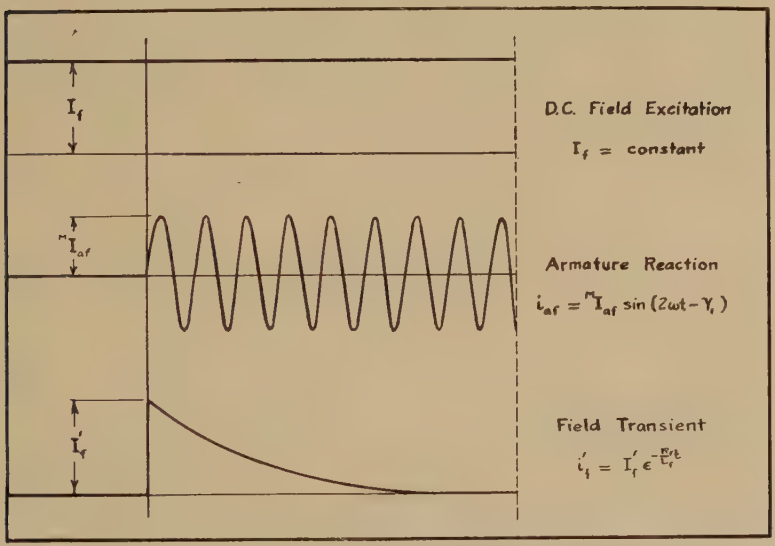

FIG. 73.—The components of the field current, V_3, in Fig. 72.

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armature field produced by the balanced armature currents, and hence the armature reaction, is constant in value and, with respect to the alternator field poles, fixed in position. In single-phase alternators the magnetic field produced by the armature currents, and therefore the armature reaction, pulsates synchronously with the armature rotation. The pulsations of the armature reaction necessarily appear in the field circuit. As the armature rotates 180 electrical

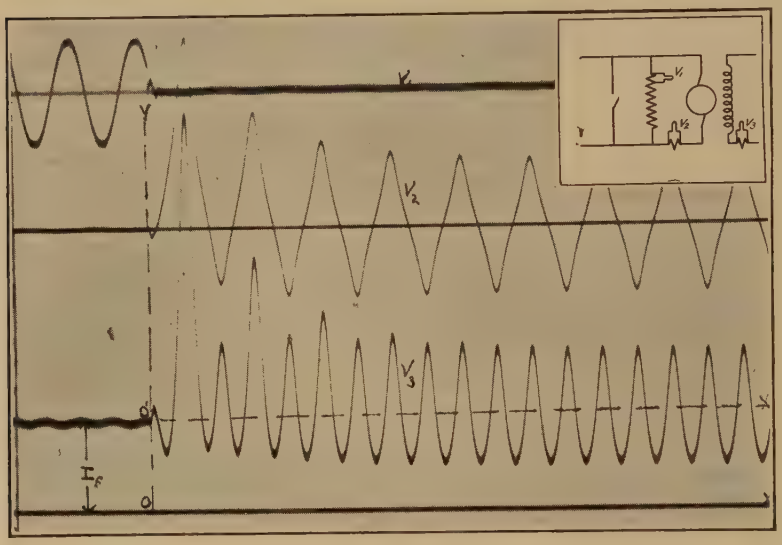


FIG. 74.—Short circuiting a single-phase alternator. No load. E = 126 volts; I_a = 12.0 amp., short circuit; I_f = 1.29 amp.; f = 60~.

degrees for each half cycle of the armature current, the pulsations of the armature reaction with respect to the field poles will have double the frequency of the armature currents. Therefore, the field current has a permanent double-frequency pulsation as shown in Figs. 71 and 72.

Since the armature reaction is pulsating and not constant, as in polyphase alternators, the initial value of the starting transient of the armature flux will depend on the point on the current wave at which the short circuit occurs. Thus in Figs. 71 and 72 the short-circuiting switch closed nearly

at the instant the armature current was zero and hence only a very small armature transient was produced. With the armature transient absent the field current-time oscillograms, as illustrated in Figs. 71 and 72, are symmetrical, showing the permanent double-frequency pulsations of the

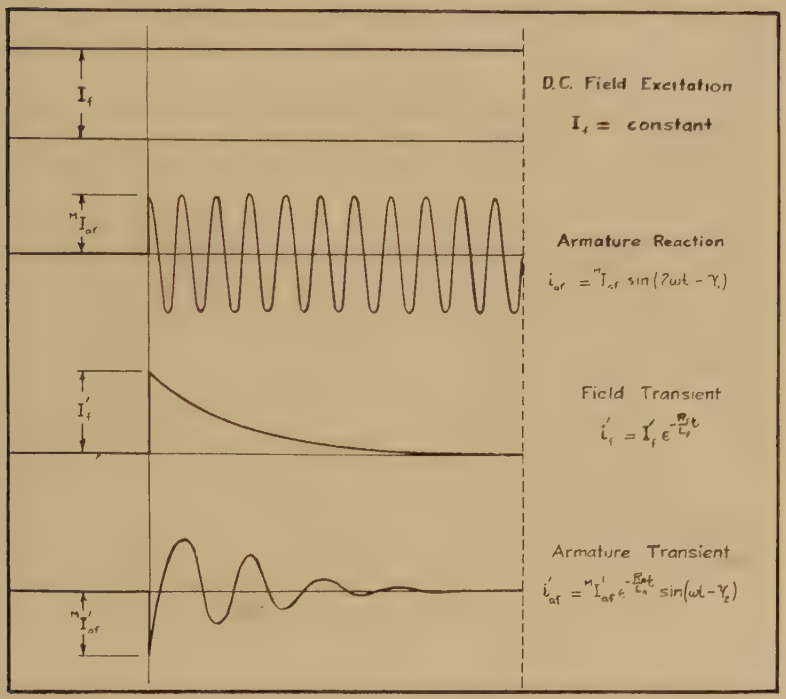

D.C. Field Excitation
I_f = constant
Armature Reaction
$i_{af} = {}^{m}I_{af} \sin(2\omega t - \gamma_1)$
Field Transient
$i'_f = I'_f \epsilon^{-\frac{R_f}{L_f} t}$
Armature Transient
$i'_{af} = {}^{m}I'_{af} \epsilon^{-\frac{R_{af}}{L_{af}} t} \sin(\omega t - \gamma_2)$

FIG. 75.—The components of the field current, V_3, in Fig. 74.

armature reaction superimposed on the field transient. The component parts of the field current-time curve in Fig. 72 are shown as separate functions in (*a*), (*b*) and (*c*) of Fig. 73. Variations due to the permeability of the iron and changes in speed of armature rotation are not shown in the equation.

If the short circuit occurs at other than the zero points on the armature current wave, an armature transient of full frequency is produced for the same reason as explained for short circuits in polyphase alternators. The oscillograms in Figs. 74 to 80 show the asymmetrical field current-time curves produced by superimposing the double-frequency permanent armature reaction, the field transient and the full-frequency pulsation produced by the armature transient upon the direct-current field excitation. The

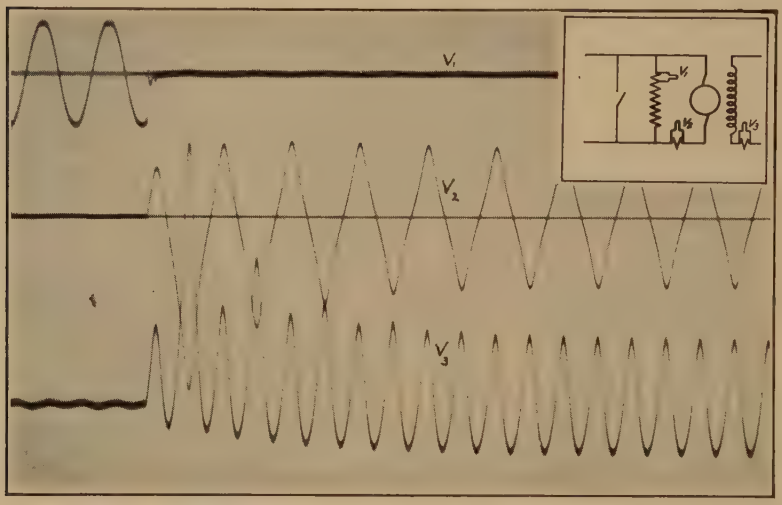


Fig. 76.—Short circuiting a single-phase alternator. No load.
E = 126 volts; I_a = 12.0 amp., short circuit; I_f = 1.29 amp.; f = 60~.

component parts of the field current in the oscillogram, Fig. 74, are shown as separate functions in Fig. 75. The distortions produced by varying permeability of the iron and in speed of armature rotation are not included in the equations. The combination of the full-frequency armature transient pulsation with the permanent double-frequency armature reaction produces the asymmetry in the curves. The ordinates for the odd numbers of the double-frequency waves add to the full-frequency values, while for the even number of waves the difference in the ordinates produces the wave recorded by the oscillograph.

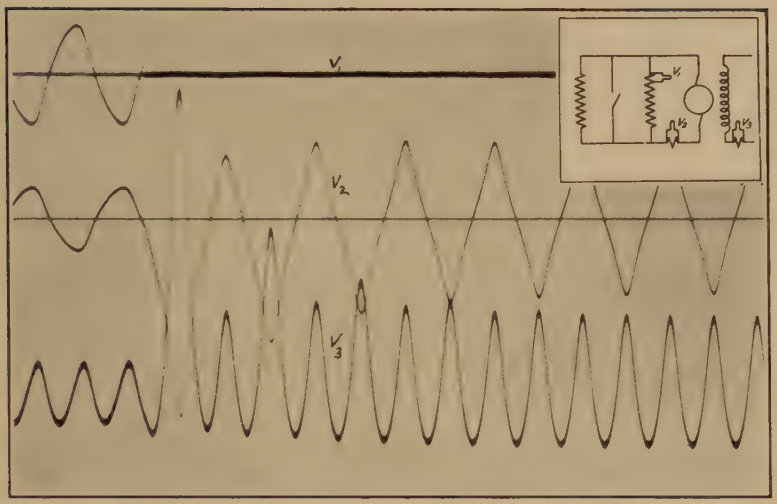


FIG. 77.—Short circuit on a single-phase alternator. Load.
E = 110 volts; I_a = 6.5 amp. with load and 13.0 amp. under short circuit; I_f = 1.39 amp.; f = 60~.

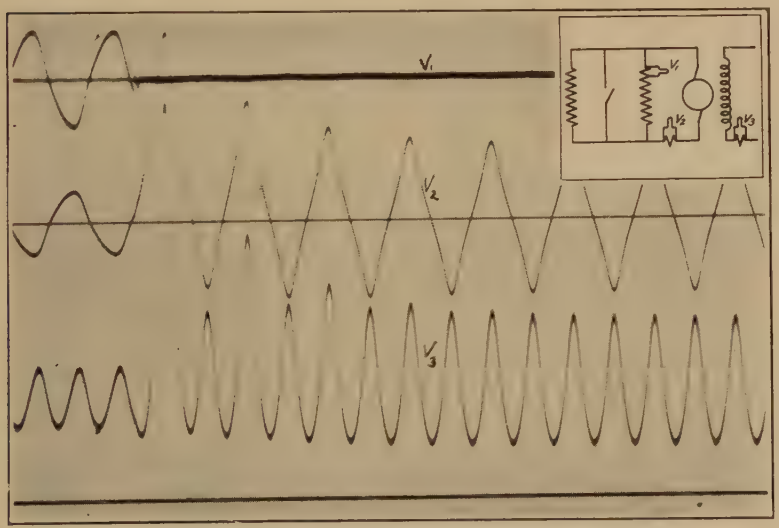


FIG. 78.—Short circuit on a single-phase alternator. Load.
E = 110 volts; I_a = 6.5 amp. with load and 13.0 amp. under short circuit; I_f = 1.39 amp.; f = 60~.

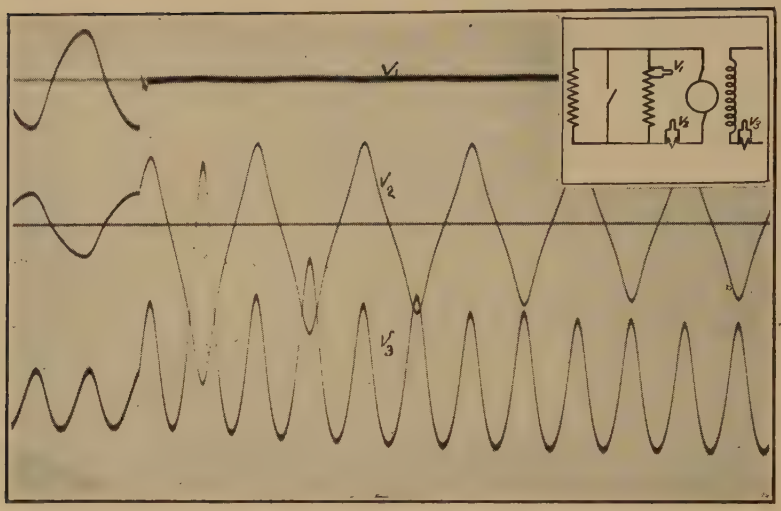


FIG. 79.—Short circuiting a single-phase alternator. Load.
E = 110 volts; I_a = 6.5 amp. with load, and 13.6 amp. on short circuit; I_f = 1.39 amp.; f = 60∼.

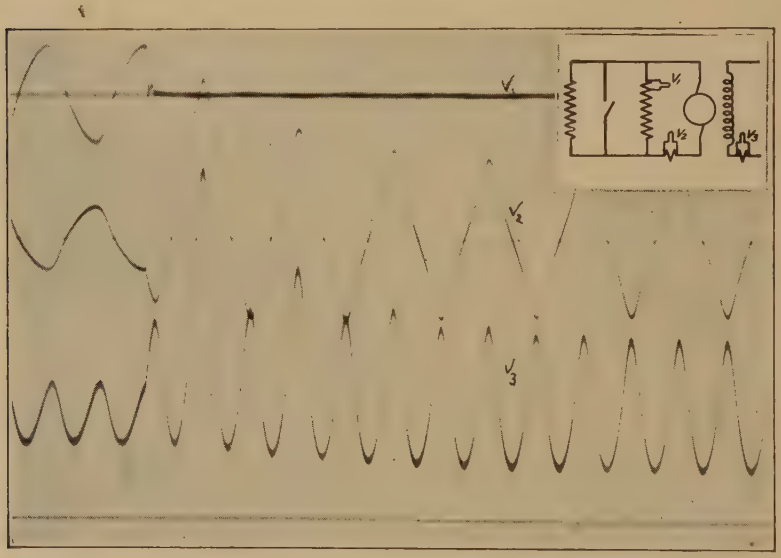


FIG. 80.—Short circuit on single-phase alternator. Load.
E = 110 volts; I_a = 6.5 amp. with load and 13.0 amp. under short circuit; I_f = 1.39 amp.; f = 60∼.

Hence, during the transition period the peaks of the odd-numbered waves decrease, while the even-numbered peaks increase, and at the expiration of the armature transient all reach the permanent constant pulsation produced by the pulsating armature reaction. While the field current pulsates as a result of the double-frequency armature reaction and the full-frequency armature transients, the voltage across the field terminals will pulsate to a greater or less degree depending on the amount of external resistance and inductance in series with the field circuit. With much external resistance or impedance the voltage at the terminals of the field winding may reach high values which may puncture the insulation and cause a short circuit in the field exciting circuit.

Under the assumption that the permeability of the magnetic circuits and the speed of rotation are constant the field current-time curve in Figs. 71 to 80 may be expressed in terms of the circuit constants and the initial values of the transients.

Let R_f = resistance of field circuit.
L_f = inductance of field circuit.
R_a = resistance of armature circuit.
L_a = inductance of armature circuit.
t = time from the instant short circuit occurs.
$\omega = 2\pi f$; f = frequency in cycles per second.
i_f = instantaneous value of field current.
I_f = permanent value of exciting current before short circuit occurs.
i_f' = instantaneous value of current in field circuit due to field transient.
I_f' = initial value of i_f'. Due to the lag, θ_a, of the armature current and the permeability factor in the field iron.
i_{af} = instantaneous value of current in field circuit due to armature reaction.
$^{m}I_{af}$ = maximum value of i_{af}.
i_{af}' = instantaneous value of current in field circuit due to armature transient.

$^{M}I'_{af}$ = maximum initial value of i'_{af}.

γ_1 = phase angle of i_{af}.

γ_2 = phase angle of i'_{af}.

$$i'_f = I'_f \epsilon^{-\frac{R_f}{L_f}t} \tag{123}$$

$$i_{af} = {}^{M}I_{af} \sin (2\omega t - \gamma_1) \tag{124}$$

$$i'_{af} = {}^{M}I'_{af} \epsilon^{-\frac{R_a}{L_a}t} \sin (\omega t - \gamma_2) \tag{125}$$

In Figs. 81, 82:

$$i_f = I_f - I'_f \epsilon^{-\frac{R_f}{L_f}t} \tag{126}$$

In Figs. 71, 72, 73:

$$i_f = I_f + I'_f \epsilon^{-\frac{R_f}{L_f}t} + {}^{M}I_{af} \sin (2\omega t - \gamma_1) \tag{127}$$

In Figs. 74, 75:

$$i_f = I_f + {}^{M}I_{af} \sin (2\omega t - \gamma_1) + I'_f \epsilon^{-\frac{R_f}{L_f}t} + {}^{M}I'_{af} \epsilon^{-\frac{R_a}{L_a}t} \sin (\omega t - \gamma_2) \tag{128}$$

As indicated by the difference in the upper and lower halves of the double-frequency pulsation, the permeability of the magnetic circuit changed with the flux density. Under full field excitation the short-circuit transients would produce much greater changes in the flux density and hence in the permeability of the steel in the armature and field poles. For this reason the equations are not directly applicable to commercial problems but state the relations of the factors involved provided the permeability of the iron core is constant.

The field transient separated from the armature reaction may be shown by taking an oscillogram of the field current when the short circuit on the single-phase alternator is broken, as shown in Figs. 81 and 82. The armature transient is dissipated during the opening of the switch, indicated on the oscillogram by the irregular breaks in the curves for a fraction of a cycle, while several complete cycles are required before the field flux, and as a consequence the armature voltage regains its full value. In the transition period following the closing or opening of the short-circuit-

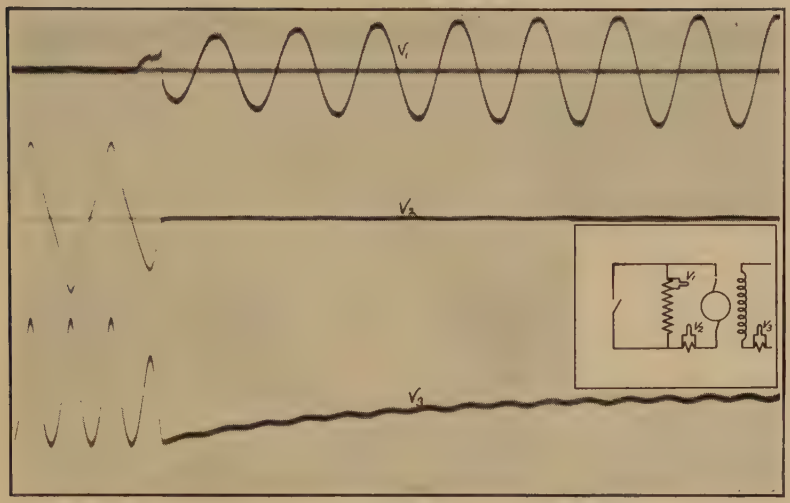


FIG. 81.—Transients on breaking short circuit. Single-phase alternator. No load.

E = 138 volts; I_a = 13.5 amp., short-circuit current; I_f = 1.42 amp.; f = 60∼.

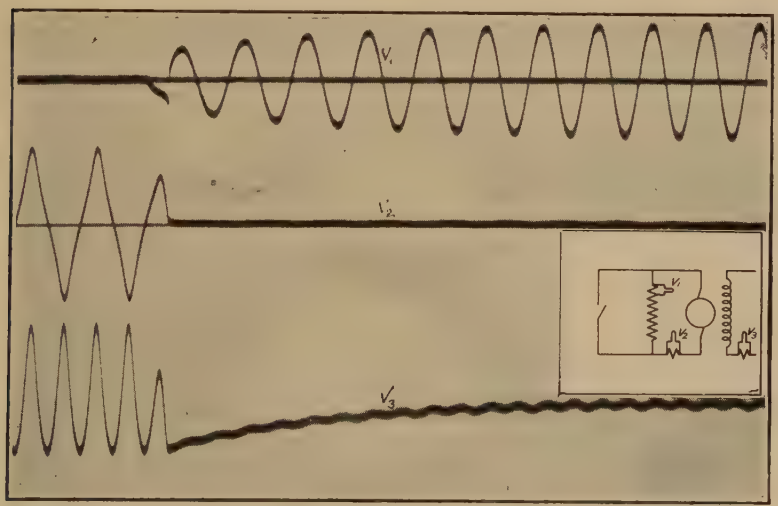


FIG. 82.—Transients on breaking short circuit. Single-phase alternator. No load.

E = 138 volts; I_a = 13.5 amp., short circuit; I_f = 1.42 amp.; f = 60∼.

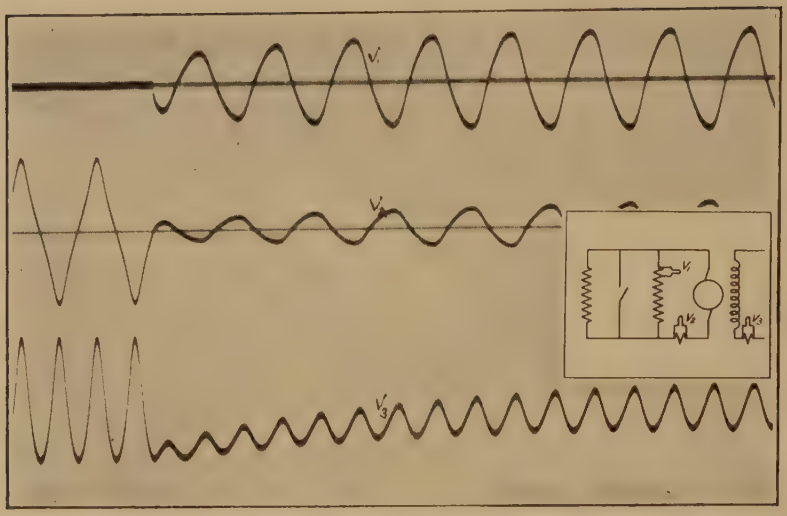


FIG. 83.—Breaking short circuit on single-phase alternator. Load.
$E = 114$ volts; $I_a = 6.8$ amp. with load and 13.5 amp. on short circuit; $I_f = 1.42$ amp.; $f = 60\sim$.

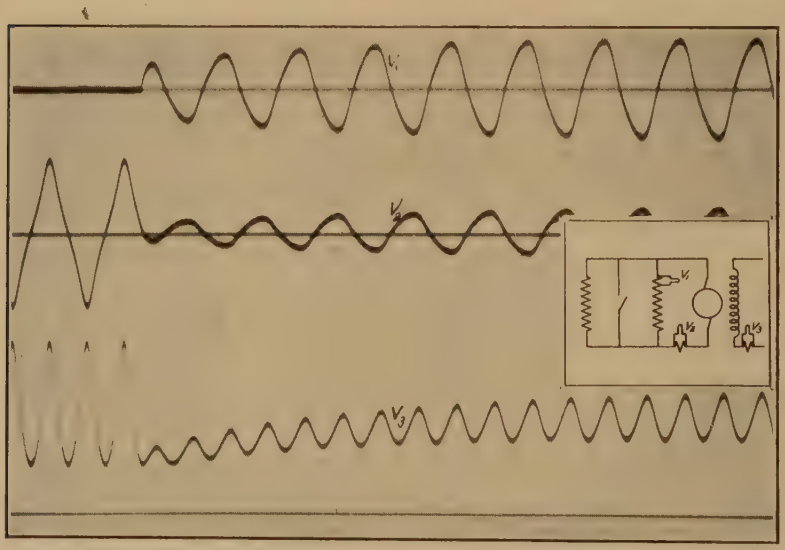


FIG. 84.—Breaking short circuit on single-phase alternator. Load.
$E = 114$ volts; $I_a = 6.8$ amp. with load and 13.5 amp. on short circuit; $I_f = 1.42$ amp.; $f = 60\sim$.

ing switch the oscillograms of the field currents show the effects of the energy changes taking place in both the field and armature circuits.

Single-phase Short Circuit on Polyphase Alternators.—If all phases of polyphase alternators are short circuited simultaneously the armature transients appear in the field circuit as full-frequency pulsations produced by the rotating magnetic field, as illustrated for three-phase machines in Figs. 63 to 66.

If one phase only is short circuited the effect on the field circuit is essentially the same as illustrated for single-phase alternators in Figs. 71 to 80. In Fig. 85 is shown the transient of the field current of a three-phase alternator after short circuiting one phase. The field current-time curve shows the effects produced by the field and armature transients and the permanent double-frequency pulsations due to the armature reaction. In Fig. 87 is shown an oscillogram for a single-phase short circuit on a three-phase alternator which after four cycles is followed by a short circuit on all three phases. While only one phase is short circuited the field current shows the double-frequency pulsations combined with both the armature and field transients. After the three-phase short circuit occurs, the field current shows the full-frequency pulsations of the armature transient combined with the slower field transient. A single-phase short circuit of short duration on a three-phase alternator is shown in Fig. 89.

Oscillograms of transients in polyphase systems produced by single-phase short circuits necessarily differ with the type of machine and the way the transient magnetic fluxes interlink with the electric circuit to which the oscillograph vibrator is connected. Thus in Fig. 90 the open-phase voltage of a two-phase alternator with short circuit on one phase shows a triple-frequency harmonic, while the field current shows the double-frequency pulsation combined with the field and armature transients of the same characteristics as for single-phase alternators.

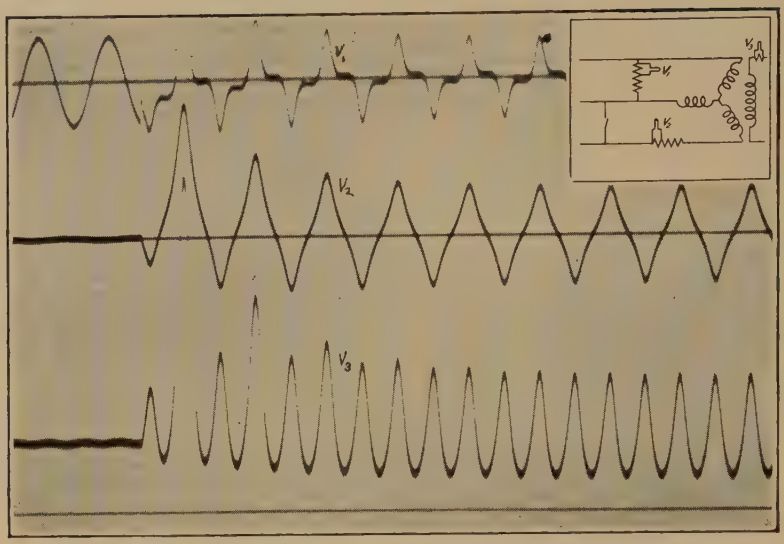


FIG. 85.—Single-phase short circuit on three-phase alternator.
E = 130 volts; I_a = 17.4 amp.; I_f = 1.58 amp.; f = 60~.

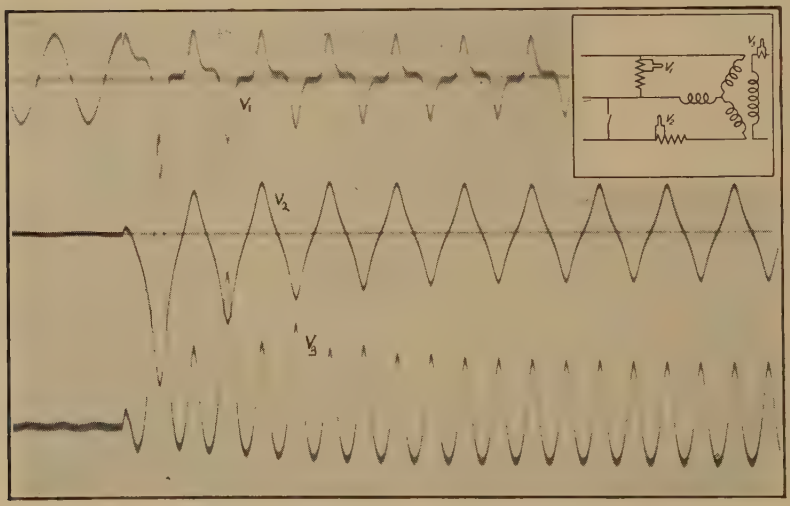


FIG. 86.—Single-phase short circuit on three-phase alternator.
E = 130 volts; I_a = 17.4 amp.; I_f = 1.58 amp.; f = 60~.

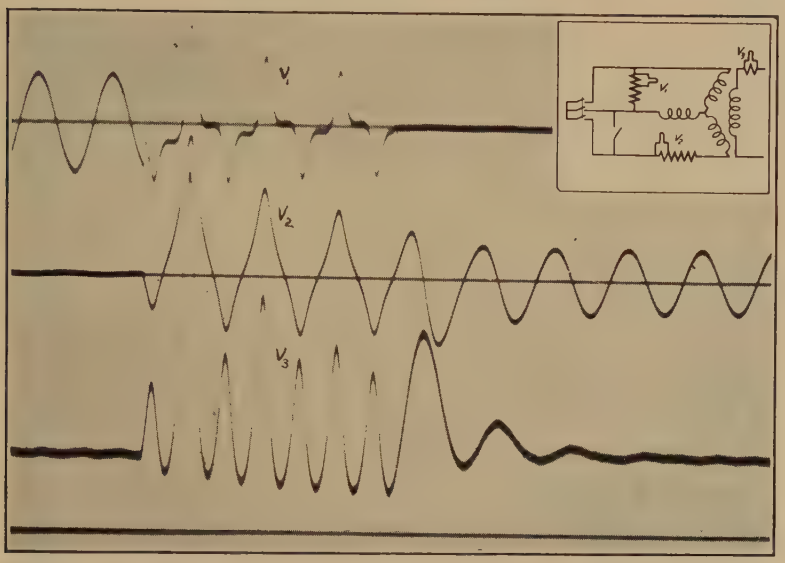


FIG. 87.—Single-phase short circuit followed by a three-phase short circuit on three-phase alternator. No load.

E = 137 volts, open circuit; I_a = 14.1 amp.; I_f = 1.60 amp.; f = 60~.

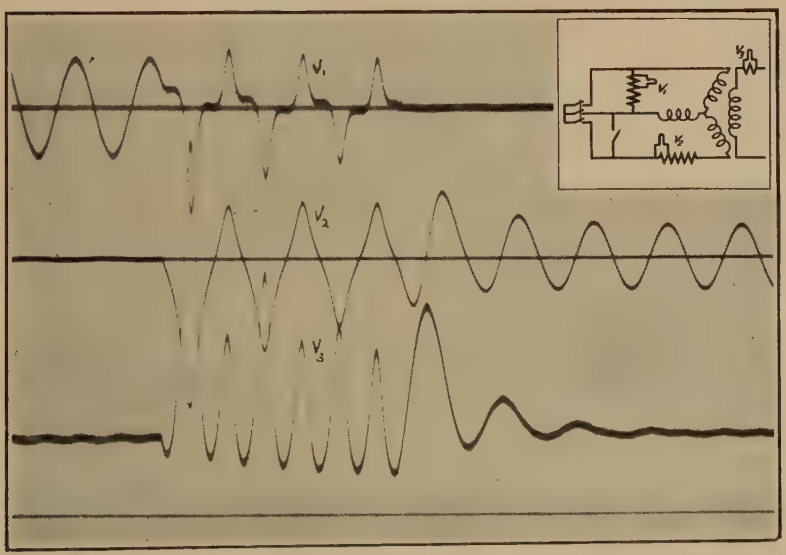


FIG. 88.—Single-phase short circuit followed by a three-phase short circuit on a three-phase alternator. No load.

E = 137 volts open circuit; I_a = 14.1 amp., short circuit; I_f = 1.60 amp. f = 60~.

FIG. 89.—Single-phase short circuit of short duration on three-phase alternator. Load.

E = 125 volts; I_a = 19.6 amp. short circuit and 5.8 amp. load; I_f = 1.58 amp.; f = 60∼.

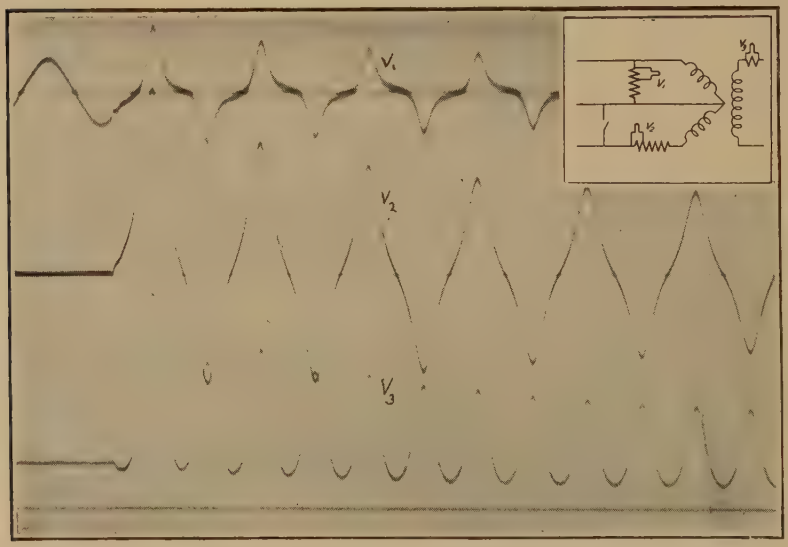


FIG. 90.—Single-phase short circuit on two-phase alternator. No load.
E = 110 volts, open circuit; I_a = 22.7 amp.; I_f = 1.57 amp.; f = 60∼.

Problems and Experiments

1. Let the sine wave curve in Fig. 91 represent the 60-cycle alternating current that would flow in a circuit having 3.0 ohms resistance, 0.05 henry inductance for a given voltage.

(*a*) Let the switch impressing the voltage on the circuit be closed at the instant marked (*a*) in the diagram. Draw in rectangular coördinates:

(1) The permanent current sine wave as in Fig. 91.

(2) The starting transient.

(3) The actual current flowing in the circuit during the first $\frac{1}{150}$ sec. after the switch is closed.

(*b*) Similar to (*a*) except the voltage is impressed at the instant marked (*b*).

2. In a circuit having 8 ohms resistance and 0.045 henry inductance a 25-cycle current is flowing, as represented by the sine wave on the left side

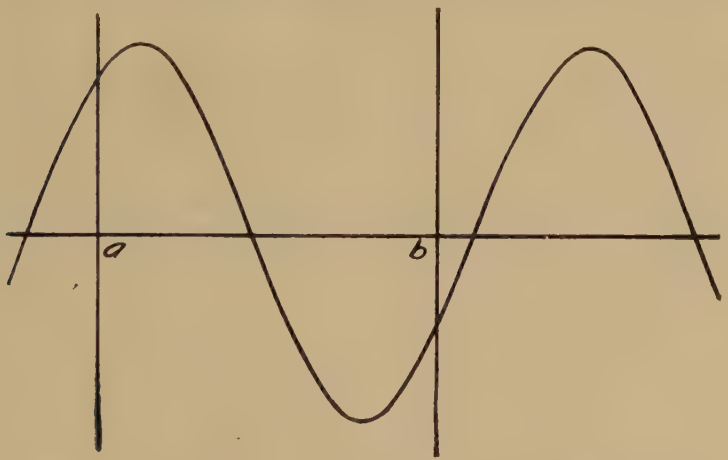


FIG. 91.—Single-phase current, sine wave, 60-cycle starting transient.

in Fig. 92. At the instant marked (*a*) the impressed voltage is suddenly changed so that it will produce a permanent 60-cycle current shown by the dotted-line sine wave in the figure.

(*a*) Draw in rectangular coördinates:

(1) The sine current waves as in Fig. 92.

(2) The starting transient.

(3) The actual 60-cycle current for the first $\frac{1}{30}$ sec. after the voltage was changed.

(*b*) Same as (*a*), except the change is made at some other point along the time axis.

3. Take an oscillogram of the starting current in a circuit of known resistance and inductance. Calculate the starting transient and draw it on the oscillogram. Check by combining the ordinates for the actual current

recorded by the oscillograph with the corresponding values of the calculated transient and compare the resulting curve with the permanent current sine wave.

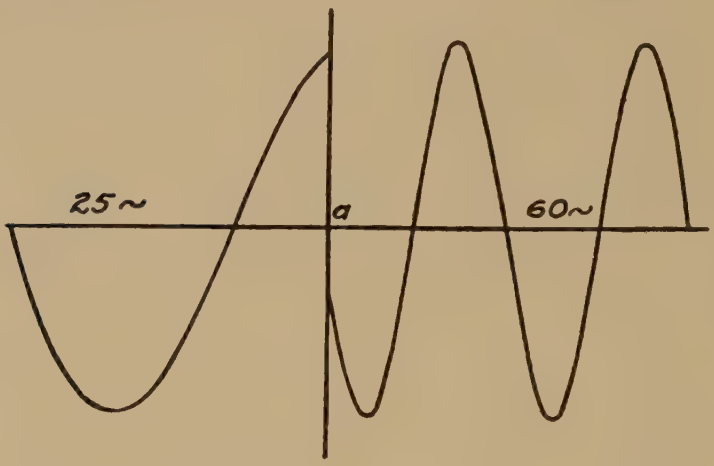


FIG. 92.—Single-phase current, sine wave, 25- to 60-cycle transient.

4. Let the sine waves in Fig. 93 represent the permanent value of the currents flowing in a balanced three-phase system, whose time constant is $\frac{1}{100}$ sec.

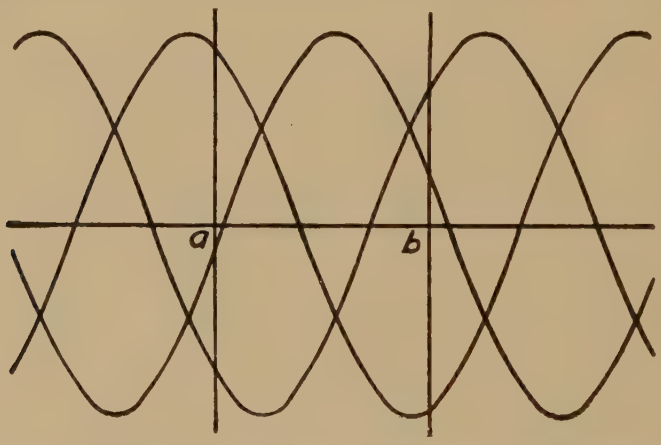


FIG. 93.—Three-phase current, sine wave, 60-cycle starting transients.

(*a*) Let the voltage be impressed at the instant marked (*a*). Draw in rectangular coördinates:

(1) The permanent current sine waves as in Fig. 93.

(2) The starting transients for the three phases.

(3) The actual currents as would be recorded by an oscillograph if a vibrator was connected to each of the three phases so as to record the current-time curves.

(*b*) Same as (*a*), except the voltage is impressed at the instant marked (*b*).

5. Take an oscillogram of the starting currents in a three-phase system connecting the vibrators as in the circuit diagram in Fig. 57. From the oscillogram and the circuit constants plot the starting transients and check with the permanent current waves as explained in problem 3.

6. Make oscillograms similar to Figs. 63 and 66 or 67 and 68. Obtain the necessary data to draw the scale in amperes or volts for each vibrator. A circuit diagram showing the position of each vibrator should be attached to each film.

7. Make oscillograms similar to Figs. 72 and 73 or 74 and 75. Quantitative data should be obtained for each vibrator and for the circuit constants.

8. Make oscillograms similar to Figs. 81 and 83 or 86 and 87 or 90.

CHAPTER V

DOUBLE-ENERGY TRANSIENTS

Single-energy transients occur in electric circuits or other apparatus in which energy can be stored in only one form. Any change in the amount of energy stored produces transients and whether the stored energy is decreased or increased the transient itself is a decreasing function with its maximum value at the first instant. In magnetic, electric and dielectric circuits in which the resistance, inductance and condensance are constant during the transition period, single-energy transients may be expressed by the exponential equation as discussed in Chaps. III and IV.

In apparatus having two forms of energy storage, as a pendulum or electric circuits having both inductance and condensance, a series of oscillations may take place by which the energy is transferred from one form to the other, while the dissipation of the stored energy into heat proceeds in much the same manner as in single-energy systems. Thus a pendulum, freely suspended in air, will swing back and forth over arcs of decreasing amplitude, with energy changing from the kinetic to the potential form and back to the kinetic twice for each cycle. The amplitude of each swing is less than for the one preceding, since part of the energy has been dissipated into heat by friction during the intervening time. The pendulum comes to rest when all the stored energy is dissipated into heat.

In electric circuits having both dielectric and magnetic storage facilities the energy stored in one form may change to the other and back and forth in a series of oscillations of definite frequency. This is illustrated by the oscillogram in Fig. 94. The energy stored in a condenser is discharged through a resistance in series with an inductance. In

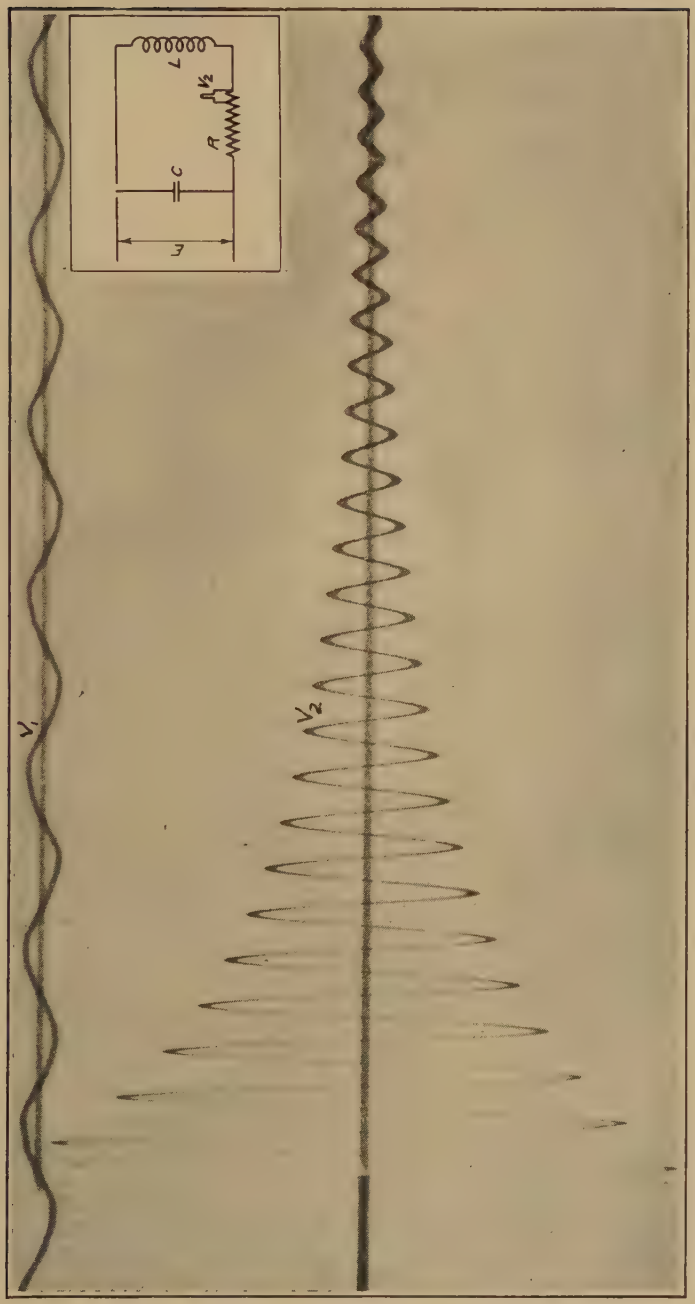


FIG. 94.—Double-energy transient.
E = 464 volts; L = 0.068 henry; C = 1.45 mf.; R = 8.68 ohms; v_1 = 100~.

passing from the dielectric field to the magnetic field or the reverse, the energy flows through the resistance and a part is dissipated by the Ri^2 losses. Hence the amplitude of each oscillation is less than for the one preceding. By referring to the timing wave on the oscillogram, Fig. 94, it is found that the frequency of oscillation was 382 cycles per second, and that practically all the energy was dissipated into heat by the Ri^2 losses in 25 cycles, or approximately $\frac{1}{15}$ sec.

Surge or Natural Impedance and Admittance.—If no energy is dissipated during the transfer the stored energy in the dielectric field when the voltage is a maximum must be equal to the quantity stored in the magnetic field when the current is a maximum. Hence from equations (7) and (16)

$$\frac{C\,{}^{\kappa}E^2}{2} = \frac{L\,{}^{\kappa}I^2}{2} \tag{129}$$

Therefore, from equation (129)

$$\frac{{}^{\kappa}E}{{}^{\kappa}I} = \sqrt{\frac{L}{C}} = {}_{\kappa}z, \text{ the } \textit{surge} \text{ or } \textit{natural} \text{ impedance of the circuit} \tag{130}$$

$$\frac{{}^{\kappa}I}{{}^{\kappa}E} = \sqrt{\frac{C}{L}} = {}_{\kappa}y, \text{ the } \textit{surge} \text{ or } \textit{natural} \text{ admittance of the circuit} \tag{131}$$

The quantity $\sqrt{L}/\sqrt{C}$ has the nature of an impedance and is called the surge or natural impedance of the circuit, and its reciprocal $\sqrt{C}/\sqrt{L}$ the natural or surge admittance of the circuit.

Frequency of Oscillation in Simple Double-energy Circuits.—Consider circuits (*a*) and (*b*) in Fig. 95. Let the inductance L, the condensance C and the resistance R be constant and of the same value in the two cases. Let an alternating-current voltage be impressed on the terminals and let the frequency be varied until the current is in phase with the voltage at the terminals. All the energy absorbed by the Ri^2 losses is supplied from the alternating-current mains.

In circuit (*a*) under the given conditions:

$$j_l x - j_c x = 0 \tag{132}$$

$$2\pi f L - \frac{1}{2\pi f C} = 0 \tag{133}$$

Hence,

$$f = \frac{1}{2\pi\sqrt{LC}} \tag{134}$$

Likewise for circuit (*b*):

$$j_c b - j_l b = 0 \tag{135}$$

$$2\pi f C - \frac{1}{2\pi f L} = 0 \tag{136}$$

Hence,

$$f = \frac{1}{2\pi\sqrt{LC}} \tag{137}$$

The expressions in equations (134) and (137) are generally used to determine the "resonance frequency" of the

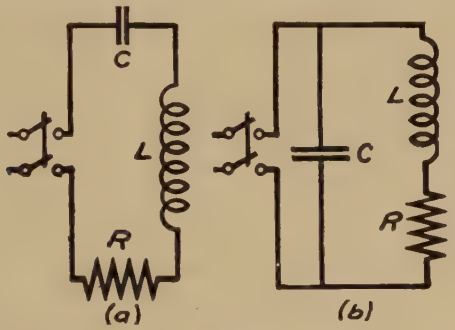


FIG. 95.—Simple series and parallel double-energy circuits.

circuits. As shown in Chap. VIII, a strict application of the definition for resonance gives a different value for the true resonance frequency unless the resistance is negligible. If all the resistance were removed from the circuits in Fig. 95 no energy would be supplied from the busbars and the stored energy would be transferred back and forth between the inductance and the condensance. With no losses the frequency of the natural or free oscillations would be the same as the "resonance frequency" given in equations (134) and (137).

In circuit (*a*), Fig. 96, and for the oscillograms in Figs 97 to 100 the condenser is charged from a direct-current supply main, after which the switch is thrown to the right so as to form an independent closed circuit with the condenser C, resistance R and inductance L in series. The energy stored in the condenser is dissipated into heat by the Ri^2 losses during a series of oscillations between the dielectric and magnetic fields.

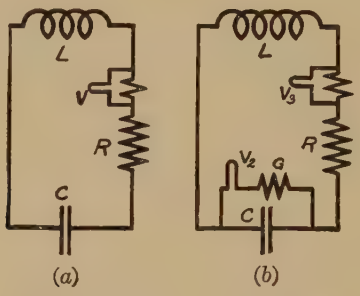


FIG. 96.—Simple oscillatory double-energy circuits.

From Kirchhoff's laws the voltage in the closed circuit, Figs. 97 to 100, while the energy originally stored in the condenser is dissipated into heat, is expressed by equations (138) or (139).

$$L\frac{di}{dt} + Ri + \frac{\int i dt}{C} = 0 \tag{138}$$

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = 0 \tag{139}$$

This is a homogeneous differential equation of the second order and its general solution is given by equation (140), in which A_1 and A_2 are the arbitrary constants.

$$i = A_1 \epsilon^{u_1 t} + A_2 \epsilon^{u_2 t} \tag{140}$$

In equation (140) if applied to Fig. 96*a*,

$$u_1 = -\frac{R}{2L} + \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}} \tag{141}$$

$$u_2 = -\frac{R}{2L} - \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}} \tag{142}$$

In order more readily to keep the dissipation or damping factors separate from the parts indicating oscillations, equations (141), (142) are rewritten in equations (143), (144):

$$u_1 = -\frac{R}{2L} + j\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \tag{143}$$

$$u_2 = -\frac{R}{2L} - j\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \tag{144}$$

From equations (140), (143), (144):

$$i = A_1\epsilon^{-\frac{Rt}{2L}}\epsilon^{j\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}}t} + A_2\epsilon^{-\frac{Rt}{2L}}\epsilon^{-j\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}}t} \tag{145}$$

But from Euler's equation for the sine and cosine:

$$\epsilon^{j\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}}t} = \epsilon^{j\omega t} = \cos\omega t + j\sin\omega t \tag{146}$$

$$\epsilon^{-j\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}}t} = \epsilon^{-j\omega t} = \cos\omega t - j\sin\omega t \tag{147}$$

Hence from equations (145), (146), (147):

$$i = A_1\epsilon^{-\frac{Rt}{2L}}[\cos\omega t + j\sin\omega t] + A_2\epsilon^{-\frac{Rt}{2L}}[\cos\omega t - j\sin\omega t] \tag{148}$$

From equations (146), (147):

$$\omega = 2\pi f = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \tag{149}$$

Hence,

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \tag{150}$$

In circuits for which the quantity under the radical is real, oscillations occur at a definite frequency as determined by equation (150) and as illustrated by the oscillograms in Figs. 97 to 100.

If the resistance,

$$R > 2\sqrt{\frac{L}{C}} \tag{151}$$

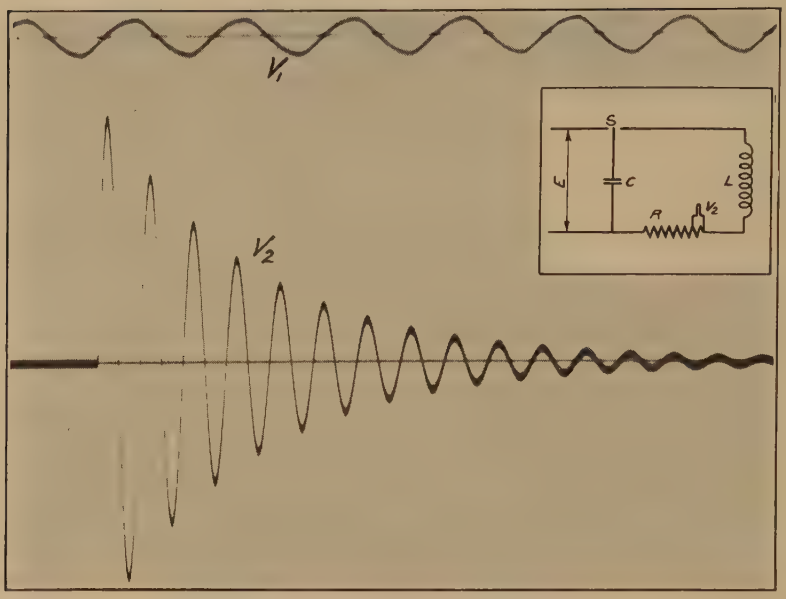


FIG. 97.—Double-energy transient.
E = 475 volts; L = 0.583 henry; C = 0.679 mf.; R = 76.5 ohms; v_1 = 100~.

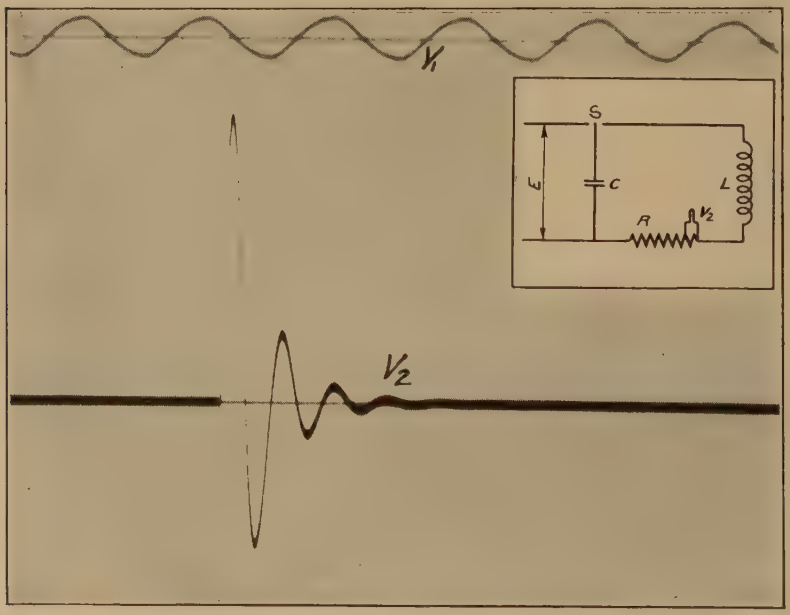


FIG. 98.—Double-energy transient.
E = 465 volts; L = 0.583 henry; C = 0.679 mf.; R = 411 ohms; v_1 = 100~.

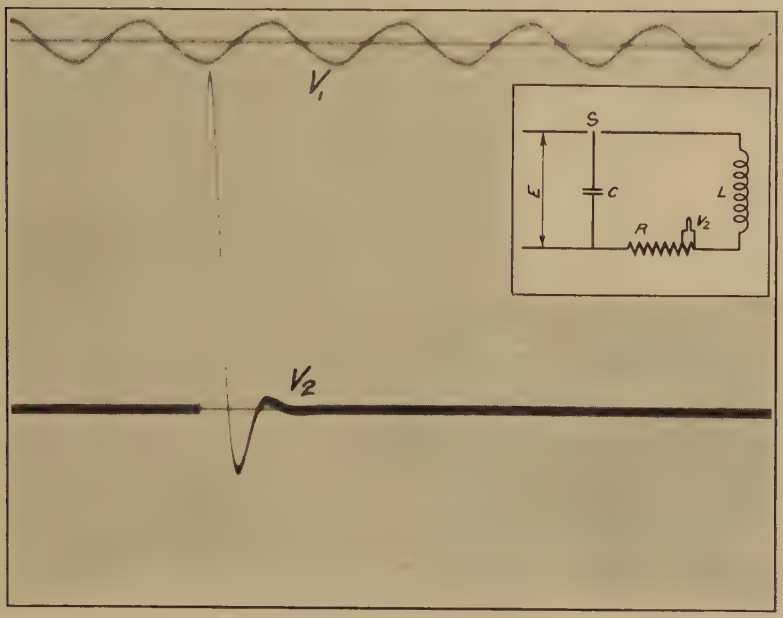


FIG. 99.—Double-energy transient.
E = 465 volts; L = 0.583 henry; C = 0.679 mf.; R = 920 ohms; v_1 = 100~.

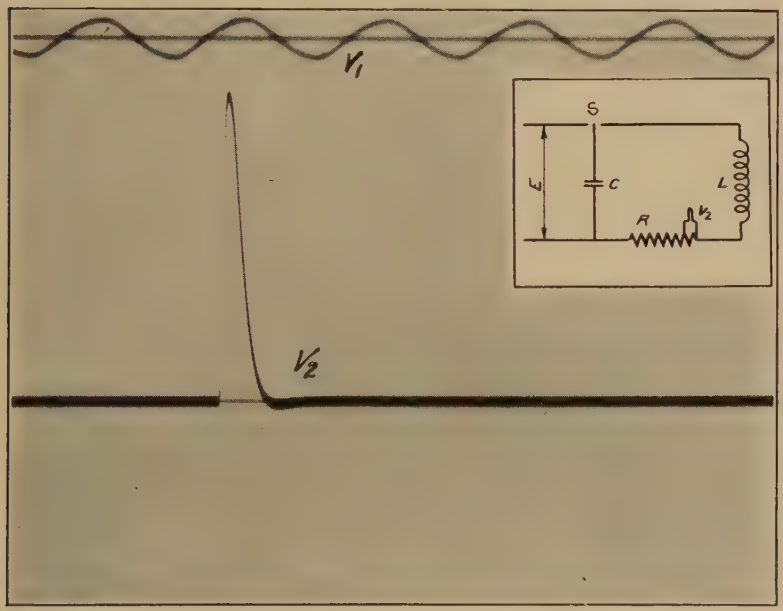


FIG. 100.—Double-energy transient.
E = 465 volts; L = 0.583 henry; C = 0.679 mf.; R = 1,643 ohms; v_1 = 100~.

the quantity under the radical sign in equation (150) becomes negative, and hence the circuit is non-oscillatory. All the energy initally stored in the condenser is dissipated into heat as the voltage and current decrease to zero. This condition is illustrated by the oscillograms in Figs. 100 and 101.

For circuits having comparatively little resistance the natural frequency of oscillations, as given in equation (150),

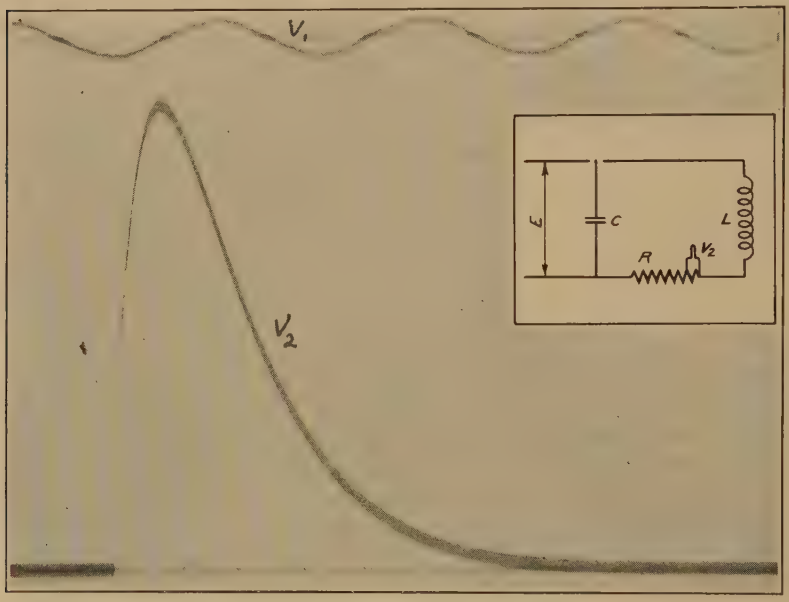


FIG. 101.—Double-energy transients.
E = 400 volts; L = 0.126 henry; C = 69.3 mf.; R = 85 ohms; $v_1 = 100\sim$

is very nearly the same as the resonance frequency given by equations (134) or (137). Thus, for the circuit data in Fig. 97 the effect of the resistance in changing the frequency of oscillation is negligible, as shown by equations (152) and (153). By increasing the resistance as in Fig. 98 and still more in Fig. 99, however, the decrease in the frequency produced by the retarding influence or drag of the resistance becomes marked. In the three cases, Figs. 97, 98 and 99, the natural or basic frequency is the same, as both C and

L are constant, but the actual frequency decreases as the resistance increases.

Resonance frequency:

$$f = \frac{1}{2\pi\sqrt{LC}} = 252.5\sim\text{(Figs. 97, 98 and 99)} \quad (152)$$

Actual frequency:

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} = 252.5\sim\text{(Fig. 97)} \quad (153)$$
$$= 245\sim\text{(Fig. 98)}$$
$$= 217\sim\text{(Fig. 99)}$$

For circuits corresponding to the conditions that would exist if the condenser in Fig. 96 were leaky, similar equations may be obtained. The voltage equation, based on Kirchhoff's laws for circuits of the type shown in Fig. 96, and in Figs. 102 to 103, is expressed by equations (154) and (160).

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{{}_ci}{C} = 0 \quad (154)$$

Using the notation shown in the circuit diagram, Fig. 96*b*, letting ${}_ce$ be the voltage across the condenser terminals, and applying Ohm's and Kirchhoff's laws

$${}_ci = i + {}_gi \quad (155)$$

$${}_gi = G\,{}_ce \quad (156)$$

$${}_ce = Ri + L\frac{di}{dt} \quad (157)$$

Hence,

$${}_ci = i + GRi + GL\frac{di}{dt} \quad (158)$$

From equations (154) and (158),

$$LC\frac{d^2i}{dt^2} + (RC + GL)\frac{di}{dt} + (1 + GR)i = 0 \quad (159)$$

or,

$$\frac{d^2i}{dt^2} + \left(\frac{R}{L} + \frac{G}{C}\right)\frac{di}{dt} + \left(\frac{1 + GR}{LC}\right)i = 0 \quad (160)$$

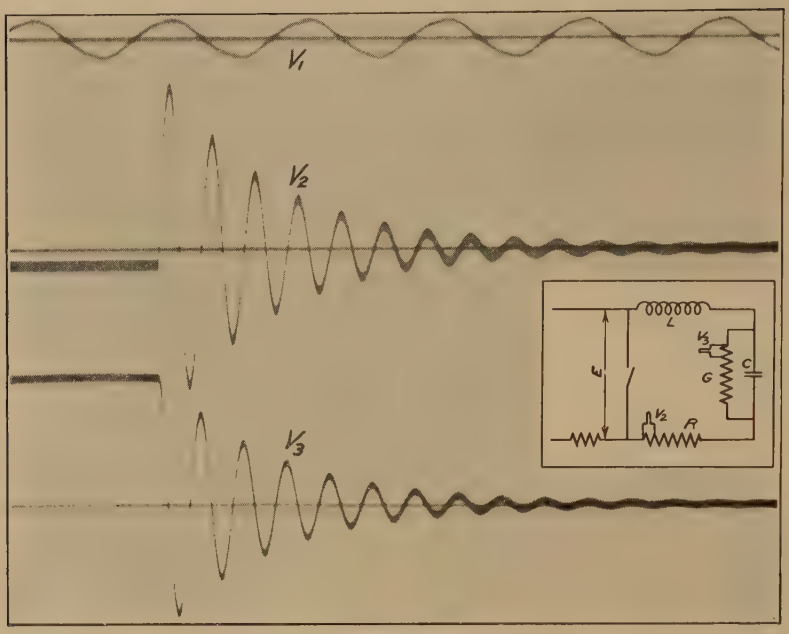


FIG. 102.—Double-energy transient.
E = 332 volts; L = 0.198 henry; C = 1.438 mf.; R = 6.36 ohms; G = 2.415 × 10^{-4} mhos; v_1 = 100∼.

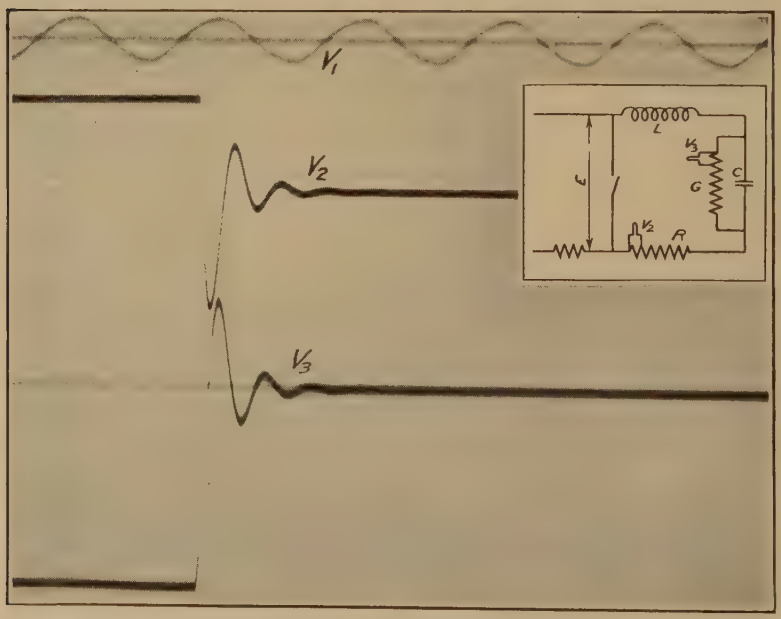


FIG. 103.—Double-energy transient.
E = 120 volts; L = 0.198 henry; C = 1.438 mf.; R = 6.36 ohms; G = 1.38 × 10^{-3} mhos; v_1 = 100∼.

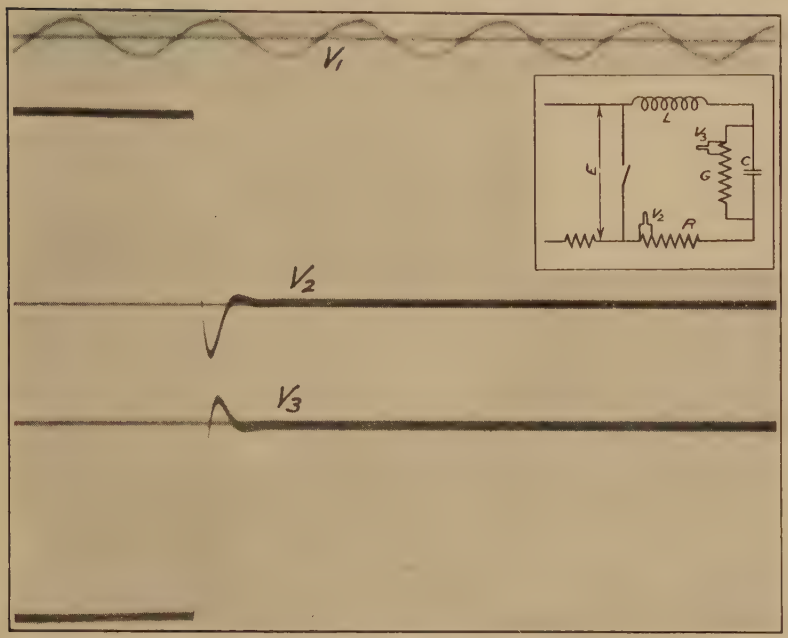


FIG. 104.—Double-energy transient.
$E = 72$ volts; $L = 0.198$ henry; $C = 1.438$ mf.; $R = 6.36$ ohms; $G = 2.9 \times 10^{-3}$ mhos; $v_1 = 100\sim$.

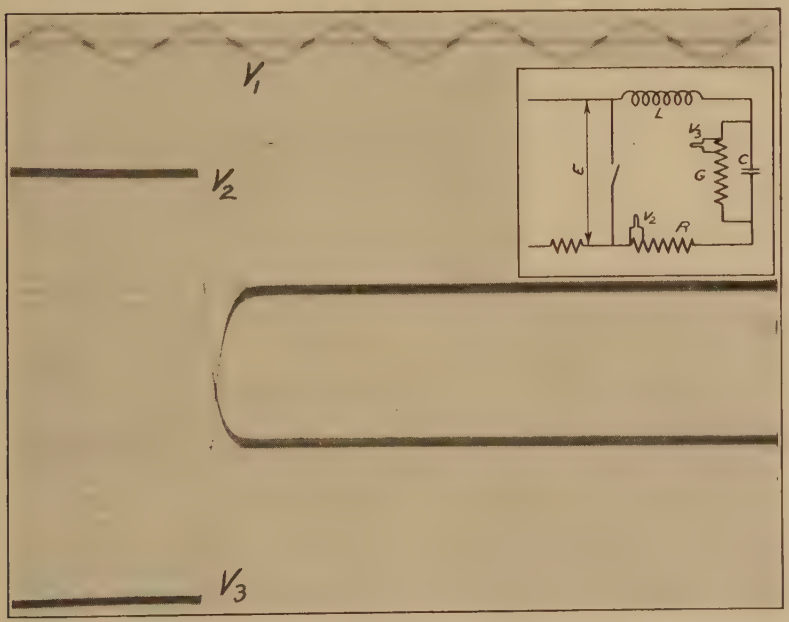


FIG. 105.—Double-energy transient.
$E = 8$ volts; $L = 0.198$ henry; $C = 1.438$ mf.; $R = 6.36$ ohms; $G = 5.5 \times 10^{-3}$ mhos; $v_1 = 100\sim$.

Equation (160) is a homogeneous differential equation of the second order of the same form as equation (139). Hence, the same general solution applies to both equations, as expressed by equation (161), in which B_1 and B_2 are the two arbitrary constants.

$$i = B_1\epsilon^{v_1 t} + B_2\epsilon^{v_2 t} \tag{161}$$

Under the given circuit conditions:

$$v_1 = -\frac{1}{2}\left(\frac{R}{L} + \frac{G}{C}\right) + \sqrt{\frac{1}{4}\left(\frac{R}{L} - \frac{G}{C}\right)^2 - \frac{1}{LC}} \tag{162}$$

$$v_2 = -\frac{1}{2}\left(\frac{R}{L} + \frac{G}{C}\right) - \sqrt{\frac{1}{4}\left(\frac{R}{L} - \frac{G}{C}\right)^2 - \frac{1}{LC}} \tag{163}$$

Rewriting equations (162), (163) so as to indicate more clearly the damping and oscillation factors, equations (164) and (165) are obtained.

$$v_1 = -\frac{1}{2}\left(\frac{R}{L} + \frac{G}{C}\right) + j\sqrt{\frac{1}{LC} - \frac{1}{4}\left(\frac{R}{L} - \frac{G}{C}\right)^2} \tag{164}$$

$$v_2 = -\frac{1}{2}\left(\frac{R}{L} + \frac{G}{C}\right) - j\sqrt{\frac{1}{LC} - \frac{1}{4}\left(\frac{R}{L} - \frac{G}{C}\right)^2} \tag{165}$$

From equations (161), (164) and (165),

$$i = B_1\epsilon^{-\frac{1}{2}\left(\frac{R}{L}+\frac{G}{C}\right)t}\epsilon^{j\sqrt{\frac{1}{LC}-\frac{1}{4}\left(\frac{R}{L}-\frac{G}{C}\right)^2}t} + B_2\epsilon^{-\frac{1}{2}\left(\frac{R}{L}+\frac{G}{C}\right)t}\epsilon^{-j\sqrt{\frac{1}{LC}-\frac{1}{4}\left(\frac{R}{L}-\frac{G}{C}\right)^2}t} \tag{166}$$

From Euler's equation,

$$\epsilon^{j\sqrt{\frac{1}{LC}-\frac{1}{4}\left(\frac{R}{L}-\frac{G}{C}\right)^2}t} = \epsilon^{j\omega t} = \cos\omega t + j\sin\omega t \tag{167}$$

$$\epsilon^{-j\sqrt{\frac{1}{LC}-\frac{1}{4}\left(\frac{R}{L}-\frac{G}{C}\right)^2}t} = \epsilon^{-j\omega t} = \cos\omega t - j\sin\omega t \tag{168}$$

Hence,

$$i = B_1\epsilon^{-\frac{1}{2}\left(\frac{R}{L}+\frac{G}{C}\right)t}[\cos\omega t + j\sin\omega t] + B_2\epsilon^{-\frac{1}{2}\left(\frac{R}{L}+\frac{G}{C}\right)t}[\cos\omega t - j\sin\omega t] \tag{169}$$

From equations (167) and (168),

$$\omega = 2\pi f = \sqrt{\frac{1}{LC} - \frac{1}{4}\left(\frac{R}{L} - \frac{G}{C}\right)^2} \tag{170}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{1}{4}\left(\frac{R}{L} - \frac{G}{C}\right)^2} \tag{171}$$

The circuit is non-oscillatory if

$$\frac{R}{L} - \frac{G}{C} > \frac{2}{\sqrt{LC}} \tag{172}$$

For circuits in which the quantity under the radical sign is greater than zero, the energy in the condenser will be dissipated into heat during a series of oscillations of definite frequency as determined by equation (171) and as illustrated by the oscillograms in Figs. 102, 103 and 104.

If the resistance and the conductance are of such values relatively to the inductance and the condensance that the quantity under the radical sign in equation (171) becomes negative, the oscillation frequency would be imaginary, that is, the circuit would be non-oscillatory. The energy initially stored in the condenser is dissipated into heat while the voltage and current decrease to zero, as illustrated by the oscillograms in Fig. 105.

For circuits in which the resistance and conductance are comparatively small, or the ratios R/L and G/C are nearly equal, the natural frequency of oscillation is very nearly the same as the resonance frequency or the natural frequency of circuits in which R and G are equal to zero. Thus for the circuit in Fig. 102 the natural oscillation frequency,

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{1}{4}\left(\frac{R}{L} - \frac{G}{C}\right)^2} = 298 \text{ cycles per sec.} \tag{173}$$

Considering R and G as negligible in determining the frequency of oscillation,

$$f = \frac{1}{2\pi\sqrt{LC}} = 298 \text{ cycles per sec.} \tag{174}$$

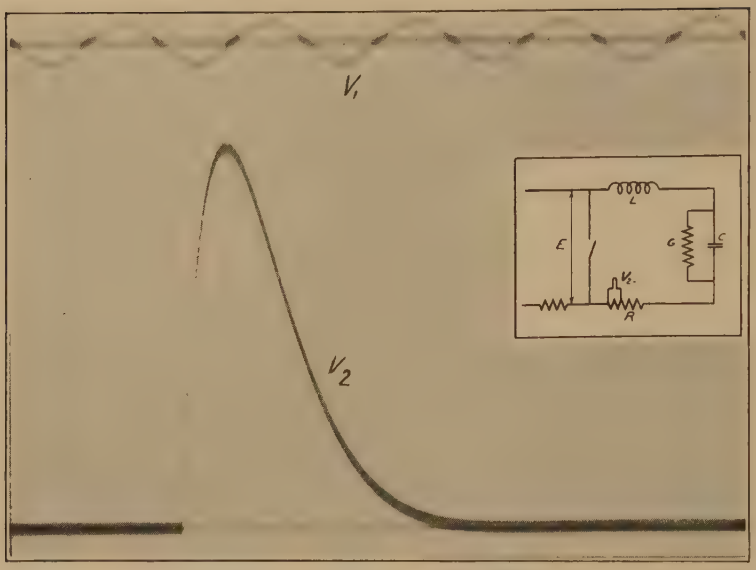


FIG. 106.—Double-energy transient.
$E = 470$ volts; $L = 0.605$ henry; $C = 18.2$ mf.; $R = 307$ ohms; $G = 0$ mho; $v_1 = 100\sim$.

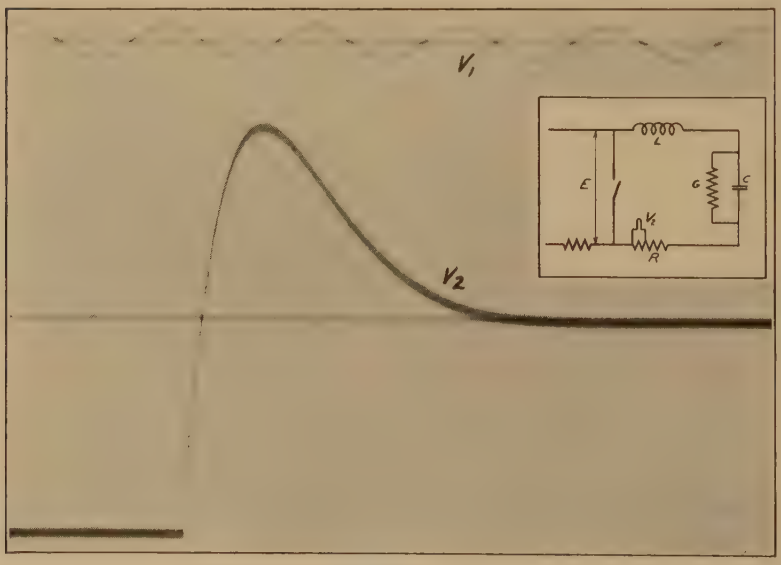


FIG. 107.—Double-energy transient.
$E = 210$ volts; $L = 0.605$ henry; $C = 18.2$ mf.; $R = 307$ ohms; $G = 8.9 \times 10^{-4}$ mhos; $v_1 = 100\sim$.

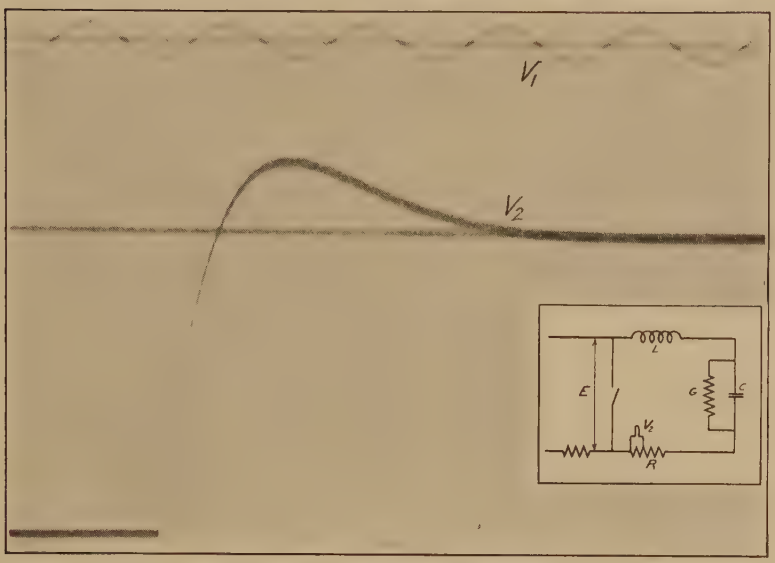


FIG. 108.—Double-energy transient.
$E = 65$ volts; $L = 0.605$ henry; $C = 18.2$ mf.; $R = 307$ ohms; $G = 4.76 \times 10^{-3}$ mhos; $v_1 = 100\sim$.

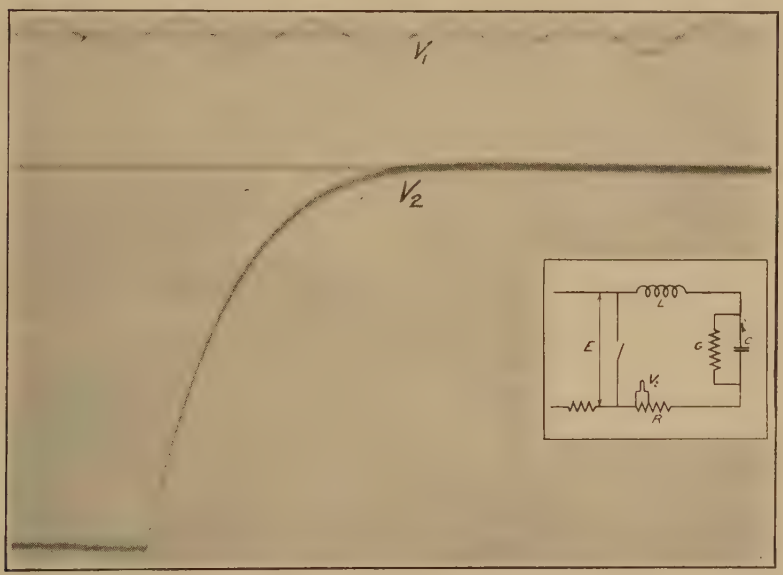


FIG. 109.—Double-energy transient.
$E = 31$ volts; $L = 0.605$ henry; $C = 18.2$ mf.; $R = 307$ ohms; $G = 1.22 \times 10^{-2}$ mhos; $v_1 = 100\sim$.

A very interesting circumstance is revealed by equation (171). A circuit having resistance greater than the critical value for oscillatory discharges as given in (172) may be made oscillatory by increasing the conductance across the terminals of the condenser without changing the resistance. This is illustrated by the oscillograms in Figs. 106, 107, 108 and 109.

For the given circuit constants in Fig. 106 the circuit is non-oscillatory. Letting R, L and C remain constant and of the same value as in Fig. 106 but increasing the conductance G, the circuit is made oscillatory in Fig. 107, although the damping factor is greater than for the circuit in Fig. 106. In Fig. 108 the oscillation was greatly reduced, and by still further increasing the conductance while R, L and C remain constant the circuit is again made non-oscillatory as shown by the oscillogram in Fig. 109.

Dissipation Constant and Damping Factor in Simple Double-energy Circuits.—In the solution for the current in double-energy circuits, Fig. 96(a) and Figs. 97 to 100, as given in equation (148), the damping factor and the dissipation constant have already been found. Similarly for circuits in Fig. 96(b) and in Figs. 102 and 108, the factors may be obtained from equation (169).

$$\text{Dissipation or attenuation constant} = \frac{1}{2}\left(\frac{R}{L} + \frac{G}{C}\right) \quad (175)$$

$$\text{Damping factor} = \epsilon^{-\frac{1}{2}\left(\frac{R}{L}+\frac{G}{C}\right)t} \quad (176)$$

In Chap. III it was shown that for single-energy transient in circuits having resistance and inductance in series the time constant, equation (97), is directly proportional to the inductance and inversely to the resistance.

$$_LT_1 = \frac{L}{R} \quad (177)$$

Similarly for circuits having condensance in parallel with conductance, the time constant, equation (49), is

directly proportional to the condensance and inversely to the conductance.

$$ {}_cT_1 = \frac{C}{G} \tag{178} $$

In double-energy circuits the energy is alternately stored in the magnetic and dielectric fields. In circuits having inductance, resistance, condensance and conductance, arranged as shown in the circuit diagrams in Figs. 102 to 105, energy is dissipated into heat both in the resistance and in the conductance. The rate of dissipation is greatest in the conductance Ge^2 when the voltage across the condenser is a maximum, that is, at the instant all the energy is stored in the dielectric field. Similarly, the rate of dissipation in the resistance Ri^2 is a maximum when the current is a maximum, that is, when all the energy is stored in the magnetic field. It is evident that since the energy is oscillating it will be in the dielectric field half of the time and in the magnetic field half of the time. Since the energy is in the dielectric form only half the actual time, the rate of dissipation in the conductance will be equal to half of what would be the case for the same circuit constants in the corresponding single-energy transient. Hence, the time constant ${}_cT_2$ for the dielectric half of the double-energy circuit would be twice the time constant ${}_cT_1$ in the corresponding single-energy transient.

$$ {}_cT_2 = 2{}_cT_1 = \frac{2C}{G} \tag{179} $$

Similarly, the time constant ${}_lT_2$ for the inductance-resistance part of the double-energy circuit would be twice the time constant ${}_lT_1$ for the corresponding single-energy transient.

$$ {}_lT_2 = 2{}_lT_1 = \frac{2L}{R} \tag{180} $$

Under the given circuit conditions with R, L, G and C constant, the proportionality law applies to double-energy

transients on the same basis as for single-energy transients. The transient term is therefore expressed by the exponential equation and appears as a factor in the current-time and voltage-time equations and represents the dissipation of energy into heat due to the resistance, Ri^2, and the conductance, Ge^2, in the circuit.

Let u represent the dissipation constant of double-energy circuits. The damping factor is, therefore,

$$\epsilon^{-ut} = \epsilon^{-\frac{t}{LT_2}} \epsilon^{-\frac{t}{CT_2}} = \epsilon^{-\frac{R}{2L}t} \epsilon^{-\frac{G}{2C}t} \tag{181}$$

$$= \epsilon^{-\frac{1}{2}\left(\frac{R}{L}+\frac{G}{C}\right)t} \tag{182}$$

$$u = \frac{1}{2}\left(\frac{R}{L} + \frac{G}{C}\right) \tag{183}$$

This is the same value as obtained in equation (169).

For circuits in which $G = 0$, as illustrated by Figs. 97 to 101, the term G/C would be zero.

Hence,

$$u' = \frac{R}{2L} \tag{184}$$

$$\epsilon^{-u't} = \epsilon^{-\frac{R}{2L}t} \tag{185}$$

This corresponds to the value of the damping factor in equation (148).

For circuits similar to Figs. 102 to 105 but in which $R = 0$, the term R/L would be zero.

Hence,

$$u'' = \frac{G}{2C} \tag{186}$$

$$\epsilon^{-u''t} = \epsilon^{-\frac{G}{2C}t} \tag{187}$$

As it is not possible completely to eliminate the resistance in circuits having inductance, the conditions for u'' can not be fully realized experimentally.

Equations for Current and Voltage Transients.—For simple double-energy circuits, with R, L and C in series,

as in Fig. 96(a), the general equation (148) for the current is

$$i = A_1\epsilon^{-\frac{Rt}{2L}}[\cos \omega t + j \sin \omega t] + A_2\epsilon^{-\frac{Rt}{2L}}[\cos \omega t - j \sin \omega t] \quad (188)$$

In equation (188) A_1 and A_2 are the arbitrary constants, which for any specific case are determined by the permanent circuit conditions preceding and following the transient period. For simple oscillations as illustrated by the oscillograms in Figs. 97, 98 and 99 only one term is used and hence for transients of this type equation (188) may be written as in equation (189), in which A_3 and A_4 are the arbitrary constants which for any specific case may be determined from the given limiting conditions under which the transient occurred.

$$i = \epsilon^{-\frac{Rt}{2L}}[A_3 \cos \omega t + A_4 \sin \omega t] \quad (189)$$

The voltage across the terminals of the condenser,

$$_ce = -Ri - L\frac{di}{dt} \quad (190)$$

From equations (189) and (190),

$$_ce = \epsilon^{-\frac{Rt}{2L}}\left\{-\frac{R}{2}[A_3 \cos \omega t + A_4 \sin \omega t] - \omega L[A_4 \cos \omega t - A_3 \sin \omega t]\right\} \quad (191)$$

For the transients in Figs. 97 to 102 the starting conditions are:

$$t = 0;\ i = 0;\ _ce = E \quad (192)$$

From equations (189), (191) and (192),

$$A_3 = 0;\ A_4 = -\frac{E}{\omega L} \quad (193)$$

Hence,

$$i = -\frac{E}{\omega L}\epsilon^{-\frac{Rt}{2L}} \sin \omega t \quad (194)$$

$$_ce = E\epsilon^{-\frac{Rt}{2L}}[\cos \omega t + \frac{R}{2\omega L} \sin \omega t] \quad (195)$$

For the given circuit constants, $\frac{R}{2\omega L}$ is very small and hence,

$$_ce = E\epsilon^{-\frac{Rt}{2L}} \cos \omega t \text{ (very nearly)} \tag{196}$$

To illustrate the application of equations (194) and (196) for the solution of numerical problems, equations (197) and (198) give the value of the current and voltage in amperes and volts for the oscillograms in Fig. 97.

$$i = -0.51\epsilon^{-65.6t} \sin (90{,}900°t) \text{ amp.} \tag{197}$$

$$e = 475\ \epsilon^{-65.6t} \cos (90{,}900°t) \text{ volts} \tag{198}$$

For the circuit in Fig. 96(*b*) the equations are of a similar nature. The general equation (169) for the current is given in equation (199), but for the oscillations shown in Figs. 102, 103 and 104 the expression may be simplified as in equation (200), in which B_3 and B_4 are constants that in each case depend on the permanent circuit conditions preceding and following the transient period.

$$i = B_1\epsilon^{-\frac{1}{2}\left(\frac{R}{L}+\frac{G}{C}\right)t}[\cos \omega t + j \sin \omega t] + B_2\epsilon^{-\frac{1}{2}\left(\frac{R}{L}+\frac{G}{C}\right)t}[\cos \omega t - j \sin \omega t] \tag{199}$$

$$i = \epsilon^{-\frac{1}{2}\left(\frac{R}{L}+\frac{G}{C}\right)t}[B_3 \cos \omega t + B_4 \sin \omega t] \tag{200}$$

$$_ce = -Ri - L\frac{di}{dt} \tag{201}$$

From equations (200) and (201),

$$_ce = \epsilon^{-\frac{1}{2}\left(\frac{R}{L}+\frac{G}{C}\right)}\left\{\frac{L}{2}\left(-\frac{R}{L}+\frac{G}{C}\right)[B_3 \cos \omega t + B_4 \sin \omega t] - \omega L[B_4 \cos \omega t - B_3 \sin \omega t]\right\} \tag{202}$$

For the transients in Figs. 102 to 104 the initial conditions are:

$$t = 0;\ i = 0;\ _ce = E \tag{203}$$

From equations (200), (202) and (203),

$$B_3 = 0; B_4 = -\frac{E}{\omega L} \tag{204}$$

Hence,

$$i = -\frac{E}{\omega L}\epsilon^{-\frac{1}{2}\left(\frac{R}{L}+\frac{G}{C}\right)t}\sin\ \omega t \tag{205}$$

$$_c e = E\epsilon^{-\frac{1}{2}\left(\frac{R}{L}+\frac{G}{C}\right)t}\left[\cos\ \omega t + \frac{1}{2\omega}\left(\frac{R}{L} - \frac{G}{C}\right)\sin\ \omega t\right] \tag{206}$$

If in equation (206),

$$\frac{R}{L} = \frac{G}{C} \tag{207}$$

$$_c e = E\epsilon^{-\frac{1}{2}\left(\frac{R}{L}+\frac{G}{C}\right)t}\cos\omega t \tag{208}$$

and

$$f = \frac{1}{2\pi\sqrt{LC}} \tag{209}$$

As an illustration of the application of equations (205) and (208) to the solution of a specific problem, let the numerical values of the circuit constants in Fig. 102 be used. Equations (210) and (211) give the instantaneous values of the current and voltage for the oscillogram in Fig. 102.

$$i = -0.9\epsilon^{-100.1t}\sin\ (107{,}100°t)\ \text{amp.} \tag{210}$$

$$_c e = 332\epsilon^{-100.1t}\cos\ (107{,}100°t)\ \text{volts} \tag{211}$$

For the oscillograms in Figs. 110 to 113 the circuits are of the same type as in Figs. 102 to 105, but the permanent conditions preceding and following the transition period are different. The imposed conditions so complicate the problem that no general solution, for R and G of any value, has been derived. Under the assumption that G is very small the voltage across the condenser terminals may be found approximately from the expression

$$_c e = E(1 - \epsilon^{-\frac{Rt}{2L}}\ \cos\ \omega t) \tag{212}$$

The oscillograms show the starting current and voltage transients at the points in the circuit indicated by the

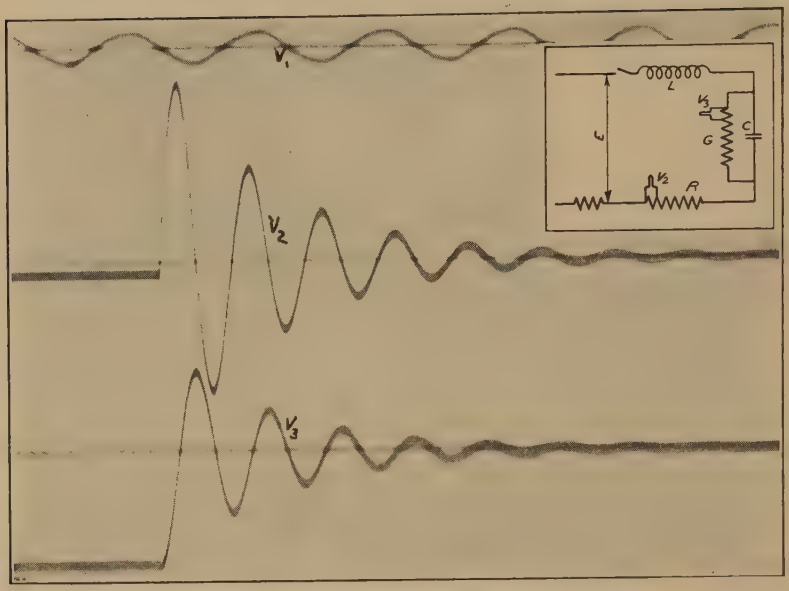


FIG. 110.—Starting current and voltage transients.
$E = 102$ volts; $R = 13.8$ ohms; $L = 0.106$ henry; $C = 8.52$ mf.; $G = 5.47 \times 10^{-4}$ mhos; $v_1 = 100\sim$.

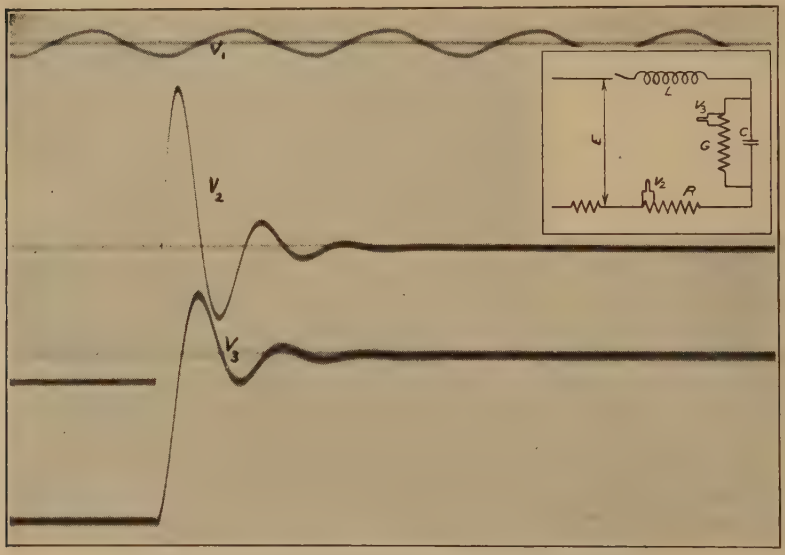


FIG. 111.—Starting current and voltage transients.
$E = 101.5$ volts; $R = 13.8$ ohms; $L = 0.106$ henry; $C = 8.52$ mf.; $G = 4.45 \times 10^{-3}$ mhos: $v_1 = 100\sim$.

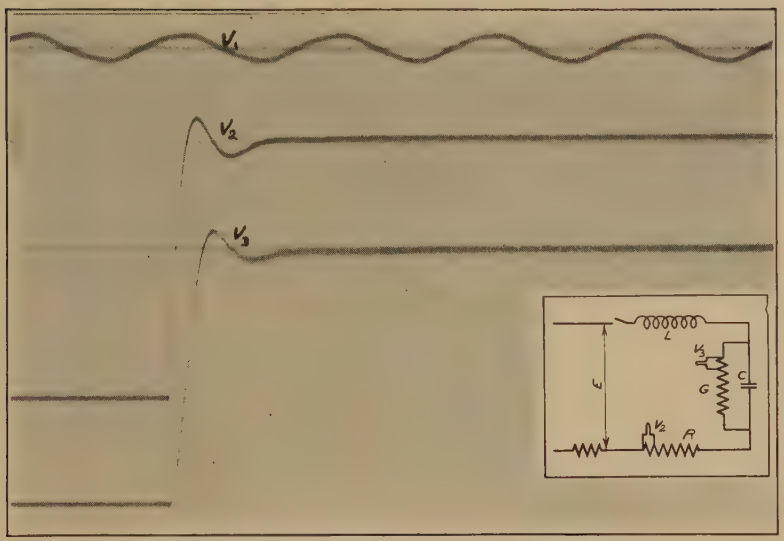


FIG. 112.—Starting current and voltage transients.
E = 101.5 volts; R = 13.8 ohms; L = 0.106 henry; C = 8.52 mf.; G = 10.7 × 10^{-2} mhos; v_1 = 100∼.

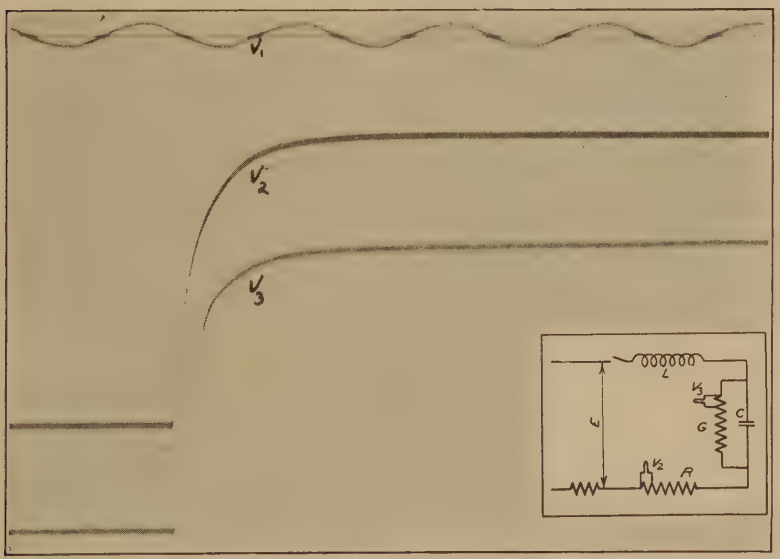


FIG. 113.—Starting current and voltage transients.
E = 101.5 volts; R = 13.8 ohms; L = 0.106 henry; C = 8.52 mf.; G = 2.3 × 10^{-2} mhos; v_1 = 100∼.

positions of the vibrators in the circuit diagram and for the values of R, L, G and C, as given in each figure. It should be noted that the voltage, Fig. 110, rises to nearly double that of the source of e.m.f. to which the circuit is connected.

Problems and Experiments

1. Given a circuit similar to Fig. 96(a) having R, L and C in series. Let $R = 20$ ohms, $L = 0.31$ henry, $C = 1.2$ mf. and $E = 120$ volts, the initital condenser discharge voltage.

(a) Find the natural period of oscillation of the circuit.

(b) Find the time constant and the damping factor.

(c) Write the equation for the transient condenser discharge current.

(d) For what values of R would the circuit be non-oscillatory.

2. Derive the equations for $_ce$, the transient voltage across the condenser terminals in Fig. 98. Trace the voltage-time curve for $_ce$ on rectangular coördinates, using the same time scale on the X-axis as in the oscillogram.

3. Take a double-energy oscillogram similar to Fig. 94. Obtain all the necessary data and write the equations for the transient current.

4. Write the equations for the voltage and current curves of the oscillogram in Fig. 103 similar to equations (210) and (211) for Fig. 102 in the text.

5. Take a series of oscillograms similar to Figs. 102 to 105. Find the values of the circuit constants and place on the film ampere and volt scales for the current and voltage curves.

CHAPTER VI

ELECTRIC-LINE OSCILLATIONS, SURGES AND TRAVELING WAVES

Electric lines, whether designed for power transmission or telephone service, may be considered as consisting of an infinite series of infinitesimal double-energy circuits of the simple types discussed in Chap. V. Each infinitesimal length of line may be represented by the resistance and inductance in one of the series circuit elements in Fig. 114 and the corresponding portion of the dielectric between the conductor and neutral by the conductance and condensance in the adjacent parallel circuit. The line constants R, L, G and C depend on the size and spacing of the conductors and the electrical properties of the dielectric and conductor materials. In order to gain clear concepts of transmission-line phenomena, it is essential for the student to conduct experiments and obtain quantitative test data. Commercial transmission lines are seldom available for experimental work, but artificial lines having the electrical characteristics of actual lines can be readily constructed of convenient design for operation in the laboratory.

Artificial Electric Lines.—Since the operating characteristics of transmission lines are determined by the line constants, the resistance, inductance, conductance and condensance, and are independent of the space and mass factors, much of the experimental work can to good advantage be performed on equivalent artificial electric lines. The oscillograms of electric-line transients used for illustrations in this chapter were obtained from an artificial transmission line, one section of which is shown in Fig. 115. This line is of the lumpy T-type of design, which

means that each unit has resistance and inductance in series combined with condensance and conductance in parallel as shown in Fig. 116. If the insulation is sufficiently high, the conductance factor may be omitted and the section circuit diagram would be as in Fig. 117, which represents the circuit diagram for the T-unit in Fig. 115.

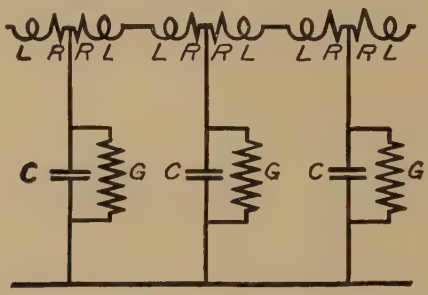


Fig. 114.—Transmission-line circuit diagram showing three elements.

In the lumpy types the line constants R, L, G and C are massed instead of uniformly distributed as in actual lines. As the lumpy type only approximates a uniform distribution of the resistance, inductance, conductance and condensance in the line, the size of each unit must be small in comparison to the total length of the line. In Fig. 115 is shown one of the forty 10-mile units in the artificial transmission line in the electrical engineering laboratories of the University of Washington. In each unit the line constants may be adjusted within the following limits:

Resistance, minimum value, 2.59 ohms.
Inductance, maximum value, 0.021 henry.
Condensance, 0.1 to 0.2 mf.

The resistance may be increased to any desired amount by moving the clamp on the resistance loop or by inserting resistance elements between the units; the inductance may be decreased by turning the right-hand coil and by taps in the lower coil; and the condensance may be varied in steps by using ten or a less number of condensers in

series. Adjustments can be made so as to give to the artificial line the electrical constants equivalent to an actual transmission line of any size of wire up to No. 0000 A.w.g. hard-drawn copper and for any spacing up to 120 in.

Fig. 115.—Section of artificial electric line. (*University of Washington.*)

The line may also be adjusted so as to be equivalent to commercial telephone lines.

Time, Space and Phase Angles.—In Chap. V the equations for the current and voltage transients were derived

for simple double-energy circuits, Fig. 96, in which the circuit constants R, L, G and C are massed. Evidently the energy transfer between the magnetic and dielectric fields would be of essentially the same nature if the inductance and resistance were intermixed with the condensance and conductance or uniformly distributed as in a transmission line. One important difference must be noted, however, which necessitates an additional factor in the expression for the transient current and voltage. In circuits having massed circuit constants the maximum value of the voltage

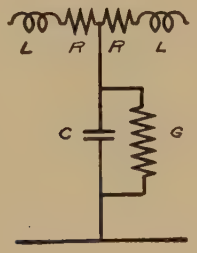


FIG. 116.—T-circuit with leaky condenser.

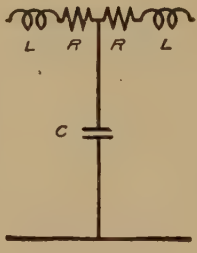


FIG. 117.—T-circuit.

will be impressed on all of the condensance at the same instant, and all parts of the magnetic field reach a maximum at the instant the current is a maximum. On the other hand, with R, L, G and C distributed, as in long transmission lines, the time required for the electric wave to travel along the length of the line enters into the problem. If a constant direct-current voltage is impressed at one end of an electric line, a short but definite time will elapse before the voltage reaches the other end of the line. If an alternating current is transmitted over the line the successive waves travel over the line at definite velocity in the same manner as the impulse from the direct-current voltage. The maximum point of any wave travels at a definite velocity as determined by the distribution of the resistance, inductance, conductance and condensance in the line. In transmission lines with air as the dielectric and with copper or aluminum conductors, the speed at which a wave or impulse

travels is slightly less than the velocity v of propagation of an electromagnetic wave in space or the velocity of light.

$$v = 3 \cdot 10^{10} \text{ cm. per sec.} \qquad (213)$$
$$= 186{,}000 \text{ miles per sec.}$$

In power transmission lines the velocity is approximately 184,000 miles per sec.

In a medium having a permeability μ and a permittivity k,

$$v' = \frac{3 \cdot 10^{10}}{\sqrt{\mu k}} \text{ cm. per sec.} \qquad (214)$$

The time required for the voltage wave to travel a distance x along the line having distributed line constants depends on the distance and velocity of propagation.

$$t_1 = \frac{x}{v} \qquad (215)$$

In comparing the transient voltage and current conditions at any two points on an electric line, x distance apart, consideration must be given to the time required for the propagation of the electric wave over the given distance and hence the time factor t must be included in the equations. In double-energy circuits having massed R, L, G and C, as in the oscillograms in Figs. 102 to 105, and for oscillations produced by the discharge of energy initially stored in the condensers, the instantaneous values of the current and voltage, under the stated conditions, are given in equations (205) and (208). By the introduction of space angles, equations of essentially the same form may be applied to circuits having distributed R, L, G and C, as illustrated by the oscillograms on transmission lines in Figs. 118 to 129.

To simplify the notations, let

$$u = \frac{1}{2}\left(\frac{R}{L} + \frac{G}{C}\right) \qquad (216)$$

$$I = -\frac{E}{\omega L} \qquad (217)$$

γ = time phase angle for $t = 0$

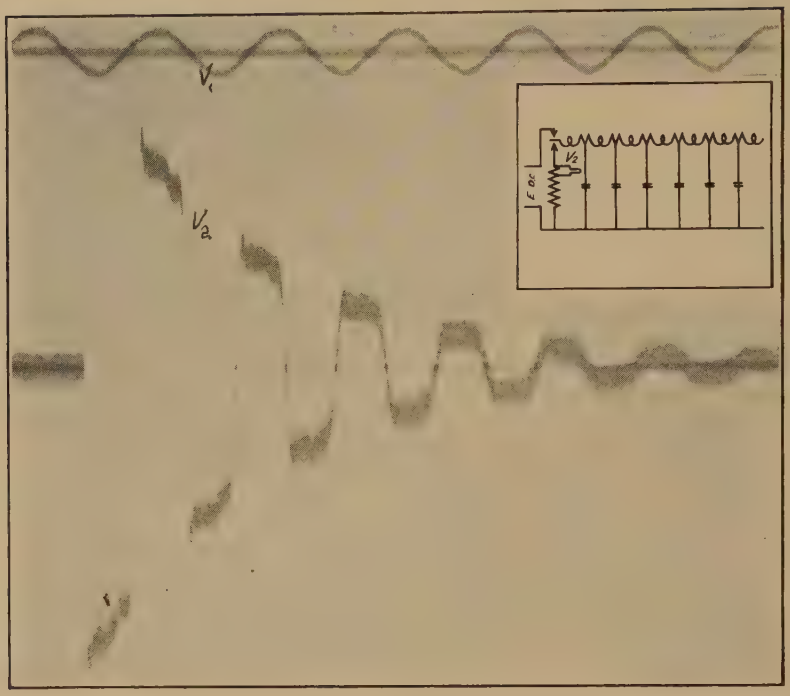


FIG. 118.—Electric-line oscillations.

E = 500 volts; L = 0.41 henry; C = 2.715 mf.; R = 50.69 ohms; vibrator v_2 resistance = 30.6 ohms; v_1 = 200∼.

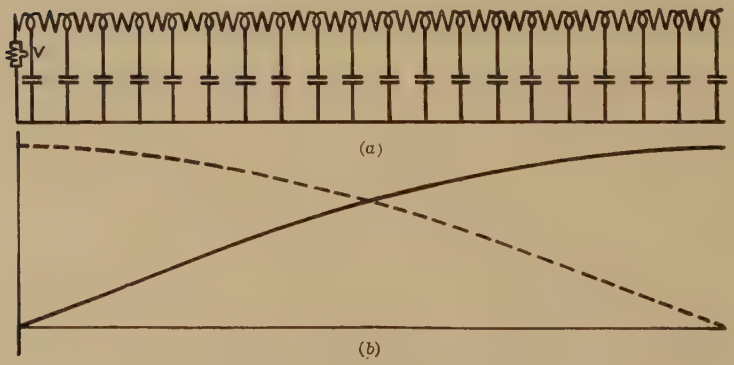


FIG. 119.—Circuit and wave diagram for Fig. 118.

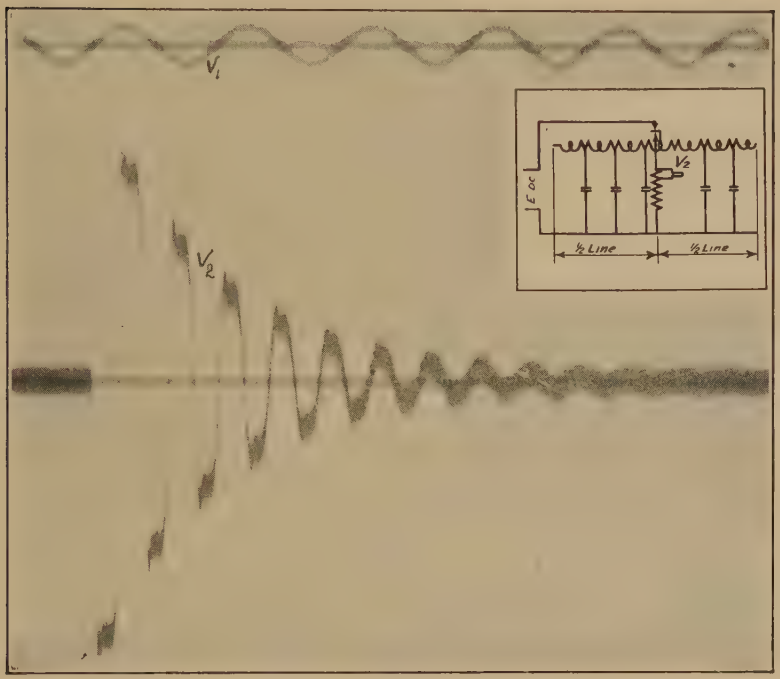


FIG. 120.—Electric-line oscillations.

E = 500 volts; L_1 = 0.205 henry; L_2 = 0.205 henry; C_1 = 1.362 mf.; C_2 = 1.362 mf.; R_1 = 25.43 ohms; R_2 = 25.26 ohms; vibrator v_2 resistance = 13.2 ohms; v_1 = 200∼.

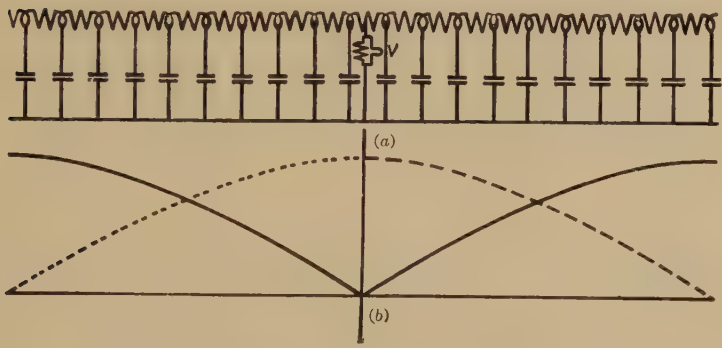


FIG. 121.— Circuit and wave diagram for Fig. 120.

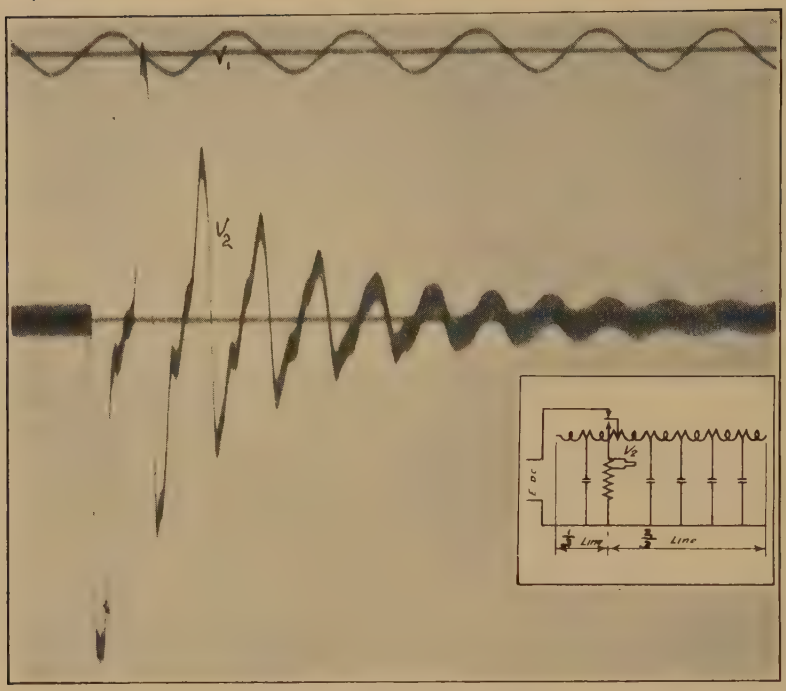


FIG. 122.—Electric-line oscillations.

E = 505 volts; L_1 = 0.104 henry; L_2 = 0.266 henry; C_1 = 0.818 mf.; C_2 = 1.636 mf.; R_1 = 15.18 ohms; R_2 = 30.39 ohms; vibrator v_2 resistance = 22.3 ohms; v_1 = 200∼.

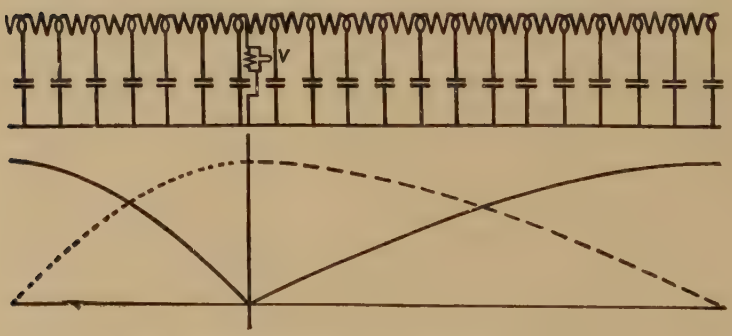


FIG. 123.—Circuit and wave diagram for Fig. 122.

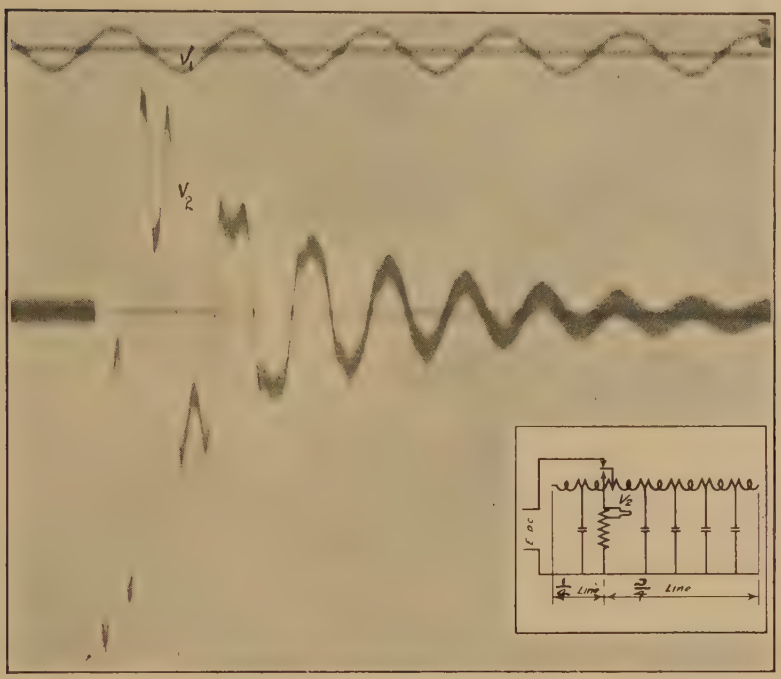


FIG. 124.—Electric-line oscillations.

E = 495 volts; L_1 = 0.083 henry; L_2 = 0.327 henry; C_1 = 0.681 mf.; C_2 = 2.044 mf.; R_1 = 12.63 ohms; R_2 = 38.06 ohms; vibrator v_2 resistance = 25.3 ohms; v_1 = 200∼

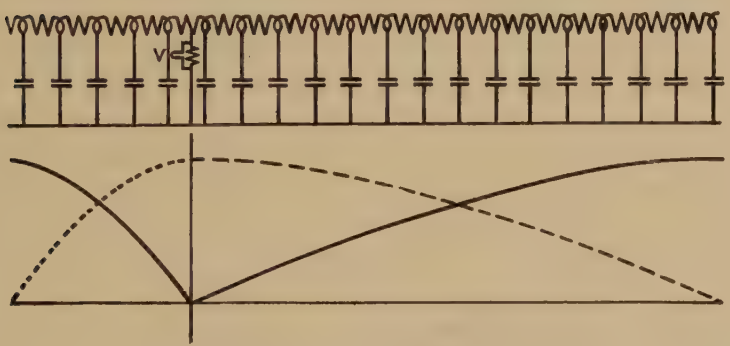


FIG. 125.—Circuit and wave diagram for Fig. 124.

For oscillations in circuits with massed R, L, G and C, under the stated assumptions:

$$i = I\epsilon^{-ut} \sin (\omega t - \gamma) \tag{218}$$

$$e = E\epsilon^{-ut} \cos (\omega t - \gamma) \tag{219}$$

For oscillations in circuit with distributed R, L, G and C, under similar conditions:

$$i = I\epsilon^{-ut} \sin [\omega(t - t_1) - \gamma] \tag{220}$$

$$e = E\epsilon^{-ut} \cos [\omega(t - t_1) - \gamma] \tag{221}$$

Substituting ϕx for ωt_1:

$$i = I\epsilon^{-ut} \sin (\omega t - \phi x - \gamma) \tag{222}$$

$$e = E\epsilon^{-ut} \cos (\omega t - \phi x - \gamma) \tag{223}$$

In equations (222) and (223) ωt is the time angle, ϕx the space angle and γ the phase angle.

Natural Period of Oscillation.—Since the space angle ϕx in equations (222) and (223) is directly proportional to the distance x from the origin, it is evident that the phase of the current i and the voltage e changes progressively along the line. At some distance l_0 the current and voltage are displaced 360° from their starting-point values. The distance l_0 is called the wave length and is the distance the electric field travels during the time t_0 required for the completion of one cycle or complete wave.

If f is the frequency of oscillations in cycles per second,

$$t_0 = \frac{1}{f} \text{ sec.} \tag{224}$$

$$l_0 = vt_0 \tag{225}$$

The fundamental frequency or natural period of free oscillation depends on the length of the line and on the imposed circuit conditions. For the oscillations recorded in the oscillogram in Fig. 118, the line is open at the receiver end and connected through the vibrator circuit at the generator end. The diagram in Fig. 119(*b*) shows that under these conditions the length of the line is one-fourth wave length of the fundamental oscillations.

In Fig. 120 the vibrator is connected at the middle point, leaving both ends open. The corresponding wave diagram in Fig. 121(*b*) shows that the length of the line is two quarter wave lengths, or one-half wave length, and the frequency of the fundamental oscillation is twice that in Fig. 118. Similarly in Fig. 122, in which the vibrator is connected at one-third the distance from one end, each of the two parts becomes a vibrating element giving fundamental oscillations. The frequency of the oscillation of the shorter part is twice as great as for the longer portion. In Fig. 124, with the vibrator connected at one-fourth the distance from one end of the line, the short end oscillates at three times the frequency of the long end. In all cases the voltage and current vary progressively along the line so that at any instant the average voltage, instead of the maximum value, is impressed on the condensers and the average current, in place of the maximum value, flows through the inductance.

The same results would be obtained in circuits with massed R, L, G and C in which the maximum voltage is impressed on all the condensance simultaneously or all of the magnetic field reaches a maximum at the instant the current is a maximum, by reducing the condensance and inductance in the ratio of the maximum to the average values. This ratio is $\pi/2$ for sine waves.

The frequency for free oscillations in simple circuits with *massed* R, L, G and C was derived in Chap. V, equation (173).

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{1}{4}\left(\frac{R}{L} - \frac{G}{C}\right)^2} \text{ cycles per sec.} \qquad (226)$$

The frequency of free oscillations in circuits having *distributed* R, L, G and C and a sine wave distribution of the voltage and current may be obtained by multiplying L and C in equation (226) by $2/\pi$, the ratio of the average to the maximum value.

$$f = \frac{1}{4}\sqrt{\frac{1}{LC} - \frac{1}{4}\left(\frac{R}{L} - \frac{G}{C}\right)^2} \text{ cycles per sec.} \qquad (227)$$

In commercial electric power lines the quantity $\frac{1}{4}\left(\frac{R}{L} - \frac{G}{C}\right)^2$ is negligibly small in comparison with $1/LC$. For practical problems the frequency of the fundamental oscillations or surges in power transmission lines with uniformly distributed R, L, G and C may therefore be obtained by equation (228).

$$f = \frac{1}{4\sqrt{LC}} \text{ cycles per sec.} \qquad (228)$$

Thus the fundamental frequency of oscillation for the transmission line in Fig. 126.

$$f = \frac{1}{4\sqrt{0.860 \cdot 5.38 \cdot 10^{-6}}} = 116.4 \text{ cycles per sec.} \qquad (229)$$

This may be checked by measurements on the oscillogram in Fig. 126. On the original film (the cut in the text is slightly reduced in size) four cycles of the timing wave (98~) measured 10 cm., while three cycles of the transient oscillations measured 6.3 cm. Hence the frequency of oscillation,

$$f = 116.6 \text{ cycles per sec.} \qquad (230)$$

Since L and C represent the total inductance and condensance of the line the frequency depends on the total length of the line or the length of time in which the oscillation occurs, as illustrated by the oscillogram in Figs. 118 or 126. The transmission line, or other circuits of distributed R, L, G and C, therefore, have a fundamental frequency at which the whole line oscillates, but as any fractional part of the line may also oscillate independently of the whole line, particularly if the oscillating section is short as compared to the entire line, oscillations of any frequency may occur. At high frequencies the successive waves are so close together that a small variation in the time constants will cause them to overlap. Since R, L, G and C are not perfectly constant, high-frequency oscillations interfere with each other, and on this account resonance phenomena occur only at low or moderate frequencies.

Length of Line.—In ordinary power transmission lines, with air as the dielectric and conductors of copper or aluminum, an electric wave or impulse travels approximately 184,000 miles per sec., about 1 per cent slower than the velocity of propagation of an electromagnetic field in free space, equation (213). In general discussions it is customary to use v as the velocity of light in the basic equations and make whatever correction may be required in the computation of any specific problem. If the length of the line is known, the frequency of the fundamental oscillation and of the harmonics can readily be determined. If the line is open at one end, the whole line is one quarter wave length of the fundamental oscillation, as illustrated by the oscillogram in Fig. 118 and corresponding diagrams in Fig. 119.

$$v = 4f_0 l_0 \tag{231}$$

Conversely, if the frequency of the oscillation is known, the length of the oscillating section may be determined. In artificial transmission lines with the frequency of the fundamental oscillation obtained from oscillograms the equivalent length of the line can be calculated. Thus from measurements on the oscillogram in Fig. 126, equation (230), $f = 116.6$ cycles per sec. Hence the length of the line,

$$l_0 = \frac{v}{4f_0} = \frac{3 \cdot 10^{10}}{4 \cdot 116.6} \text{ cm.} = 643.7 \text{ km.} = 400 \text{ miles} \tag{232}$$

From equations (213), (228) and (231), relations are obtained by which L or C may be calculated if the length of the line l in cm. and either C or L are known.

$$v = 3 \cdot 10^{10} = 4fl = \frac{l}{\sqrt{LC}} \tag{233}$$

Hence,

$$L = \frac{l^2}{v^2 C} \text{ or } C = \frac{l^2}{v^2 L} \tag{234}$$

For cables or circuits in which the permeability μ and the permittivity κ are greater than unity the corresponding

relations are obtained from equations (214), (228) and (231).

$$v' = \frac{3 \cdot 10^{10}}{\sqrt{\mu\kappa}} = 4fl = \frac{l}{\sqrt{LC}} \tag{235}$$

$$L = \frac{l^2}{v'^2 C} \text{ or } C = \frac{l^2}{v'^2 L} \tag{236}$$

These equations are useful in the calculation of the condensance of circuits in which the inductance can be more easily determined, as in complex overhead systems, and in calculating inductance in cables or other circuits in which the condensance may readily be measured.

Velocity Unit of Length. Surge Impedance.—In handbooks and tables the values of R, L, G and C are given for some unit of length as a centimeter, kilometer, 1,000 ft., mile, etc. In discussions and calculations of transient phenomena the velocity unit of length is sometimes used. For overhead structures the unit of length l on this basis would be v, or $3 \cdot 10^{10}$ cm. Hence from equation (234), and under the assumptions made in its derivation,

$$L_v = \frac{1}{C_v} \tag{237}$$

The natural or surge impedance from equations (130) and (237):

$$z_v = \sqrt{\frac{L_v}{C_v}} = L_v = \frac{1}{C_v} \tag{238}$$

$$y_v = \sqrt{\frac{C_v}{L_v}} = C_v = \frac{1}{L_v} \tag{239}$$

By the use of the velocity unit of length investigations on transmission systems having sections of different constants and hence of different wave length are greatly simplified. In systems having overhead lines, cables, coiled windings, as in transformers, arresters, etc., the wave length becomes the same in the velocity measure of length.

Voltage and Current Oscillations and Power Surges.—It has been shown that in free or stationary oscillation transmission lines or other electric circuits having uniformly distributed R, L, G and C the current and voltage are essentially in time quadrature. From equations (240) and (241):

$$i = I\epsilon^{-ut} \sin(\omega t - \phi x - \gamma) \tag{240}$$

$$e = E\epsilon^{-ut} \cos(\omega t - \phi x - \gamma) \tag{241}$$

Hence, the instantaneous power p at any point in the circuit is given by equation (242):

$$p = ei = \frac{EI}{2}\epsilon^{-2ut} \sin 2(\omega t - \phi x - \gamma) \tag{242}$$

The direction of the flow of power changes $4f$ times each second, since the sine function becomes alternately positive and negative for successive π time degrees. That is, a surge of power occurs in the circuit of double the frequency of the current or voltage oscillations, although the average flow of power along the line is zero.

$$\text{Average power } p_0 = 0 \tag{243}$$

General Transmission-line Equations.—In the preceding paragraphs various phases of the electric transients that occur during the free or natural oscillations of electric circuits have been discussed. The general problem, in which transient phenomena occur while continuous power is supplied to the system and transmitted along the line, is necessarily much more complex. In transmission lines or other electric circuits having uniformly distributed resistance, inductance, conductance and condensance, with R, L, G and C the constants per unit length of line, the voltage and current relations in time may be expressed by partial differential equations as in equations (245) and (246):

$$\frac{\partial e}{\partial x} = L\frac{\partial i}{\partial t} + Ri \tag{245}$$

$$\frac{\partial i}{\partial x} = C\frac{\partial e}{\partial t} + Ge \tag{246}$$

Differentiating equation (245) with respect to x and equation (246) with respect to t and eliminating $\frac{\partial^2 i}{\partial x \partial t}$, equations (247) and (248) may be derived:

$$\frac{\partial^2 e}{\partial x^2} = LC\frac{\partial^2 e}{\partial t^2} + (RC + GL)\frac{\partial e}{\partial t} + RGe \qquad (247)$$

$$\frac{\partial^2 i}{\partial x^2} = LC\frac{\partial^2 i}{\partial t^2} + (RC + GL)\frac{\partial i}{\partial t} + RGi \qquad (248)$$

A general solution for these equations is given in equation (249), one term of which represents the sum of the outgoing and the other the sum of the incoming waves.

$$e = A_1\epsilon^{\pm at}\,\epsilon^{bx}\cos(\alpha t \mp \beta x - \gamma_1) + A_2\epsilon^{\pm at}\,\epsilon^{-bx}\sin(\alpha t \mp \beta x - \gamma_2) \qquad (249)$$

In order to determine the values of A_1, A_2, a, b, α, β, γ_1 and γ_2, the specific conditions under which the line operates must be given. It is, however, of first importance to understand the purpose or functions of each term in the equation. On the basis of energy flow and dissipation in a line transmitting power, the following interpretation of the symbols in equation (249) may be helpful.

A_1 and A_2 are constants whose values are determined by the limiting conditions of each specific problem.

ϵ^{-at} may be called the *time damping factor* and a the *time dissipation* constant for the transient oscillations. This factor represents the same form of energy dissipation as $\epsilon^{-\alpha t}$ in Chap. V. Ordinarily the transformation of electric energy into heat by the Ri^2 and Ge^2 losses is non-reversible and therefore the sign of the dissipation constant must be negative.

$\epsilon^{\pm bx}$ may be called the *distance damping factor* and b the *distance dissipation constant.* It relates both to the losses along the line in the steady flow of energy, as in transmission lines carrying permanent load, and to the flow of transient energy in the system as with traveling waves or in the oscillations of compound circuits.

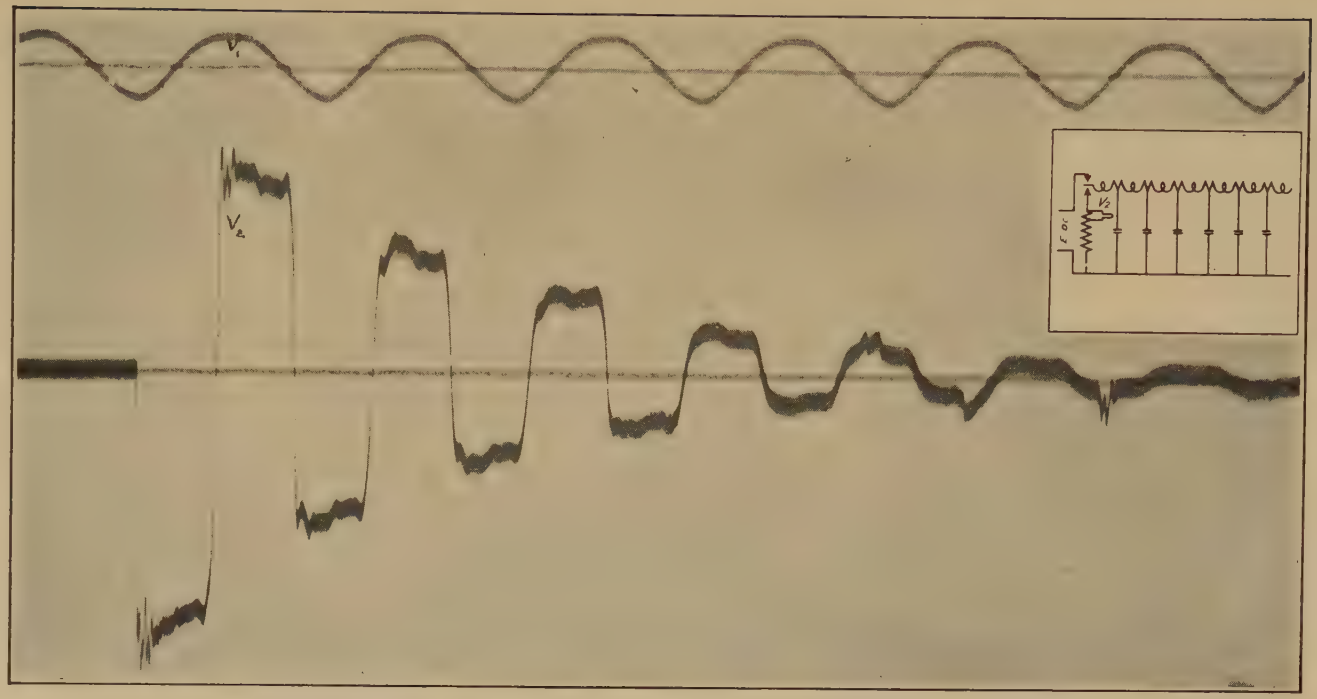


FIG. 126.—Electric-line oscillations. Receiver end open. Oscillograph at generator end.
E = 500 volts; L = 0.860 henry; C = 5.38 mf.; R = 100 ohms; $v_1 = 98\sim$. Equivalent length, 400 miles; 4/0 copper.

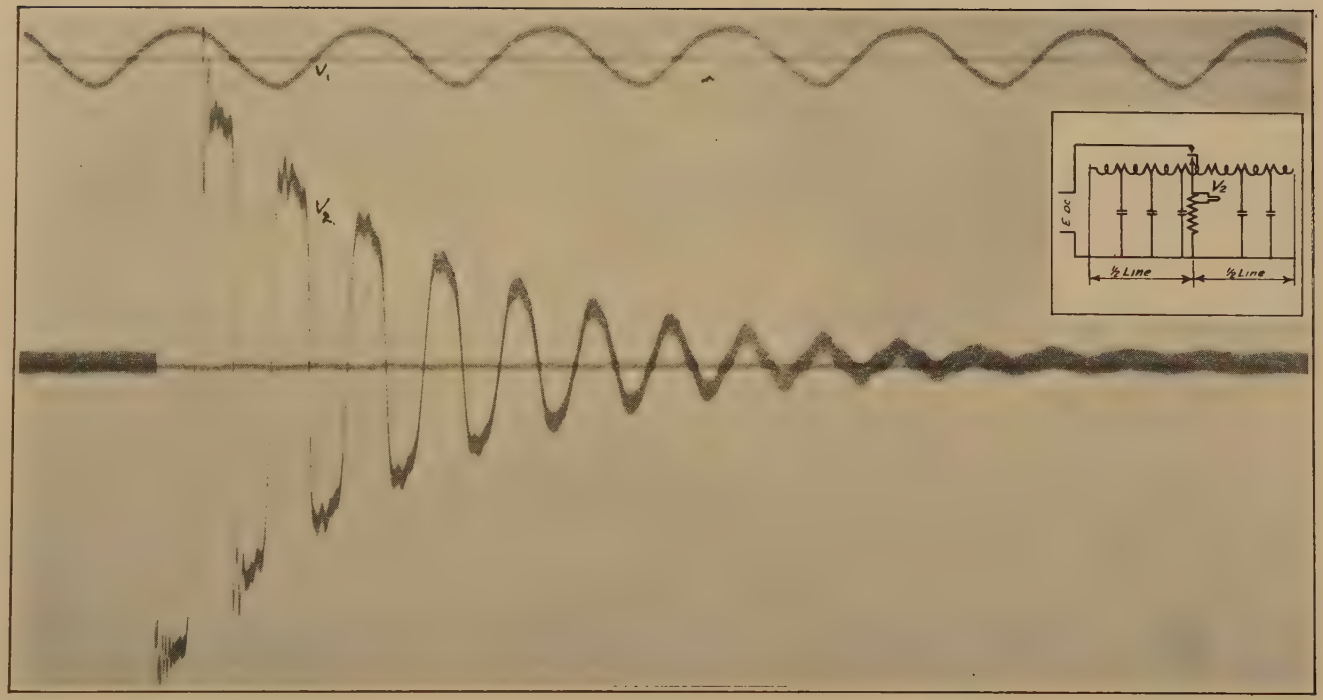


FIG. 127.—Electric-line oscillations. Both ends open. Oscillograph at middle point of line.
E = 500 volts; L = 0.860 henry; C = 5.38 mf.; R = 100 ohms; v_1 = 100〜. Equivalent length, 400 miles; 4/0 copper.

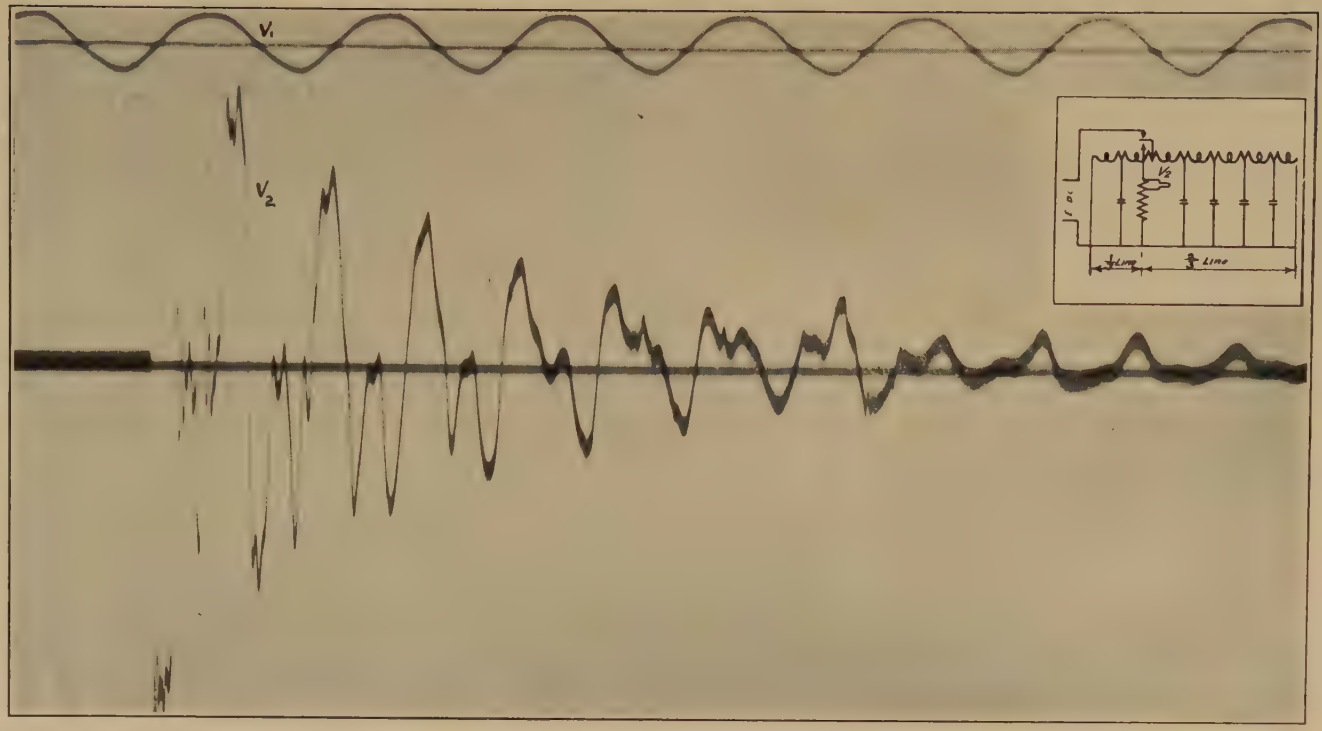


FIG. 128.—Electric-line oscillations. Both ends open. Oscillograph at one-third length of line from generator end. E = 500 volts; L = 0.860 henry; C = 5.38 mf.; R = 100 ohms; v_1 = 100∼. Equivalent length, 400 miles; 4 0 copper.

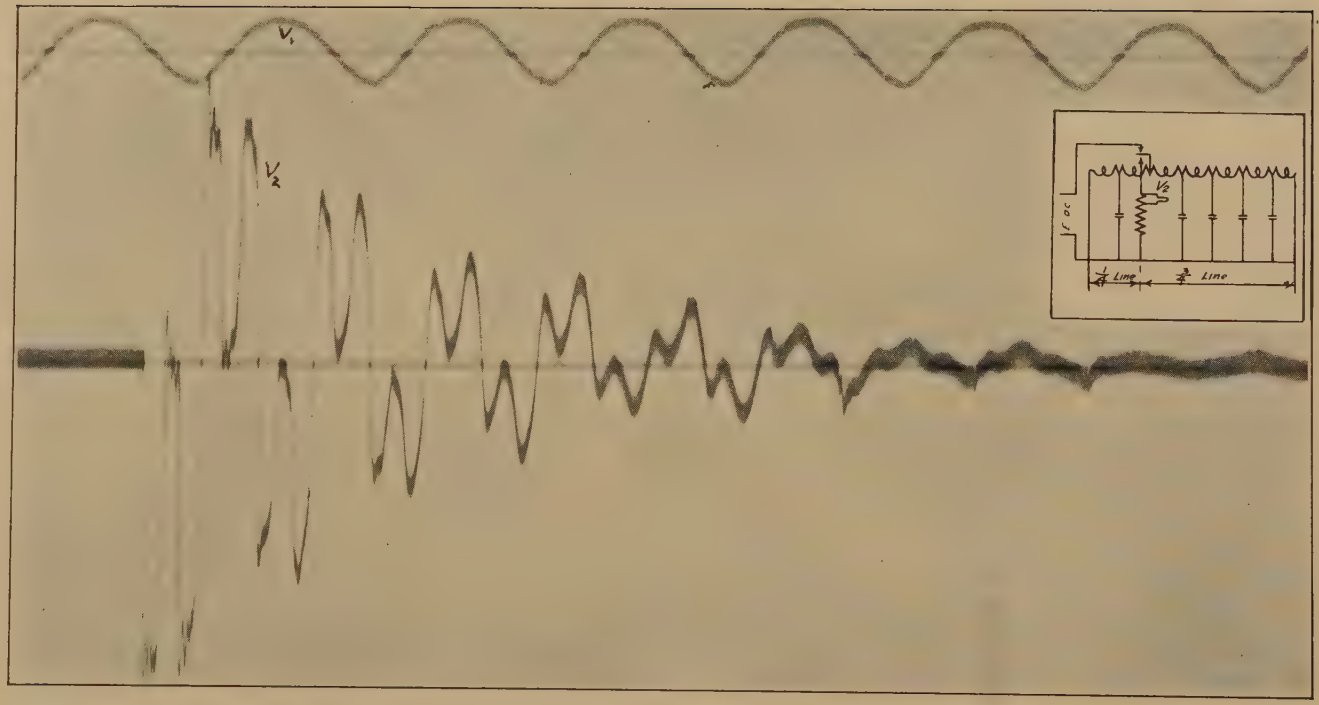


FIG. 129.—Electric-line oscillations. Both ends open. Oscillograph at one-fourth length of line from generator end. E = 500 volts; L = 0.860 henry; C = 5.38 mf.; R = 100 ohms; $v_1 = 100\sim$. Equivalent length, 400 miles; 4/0 copper.

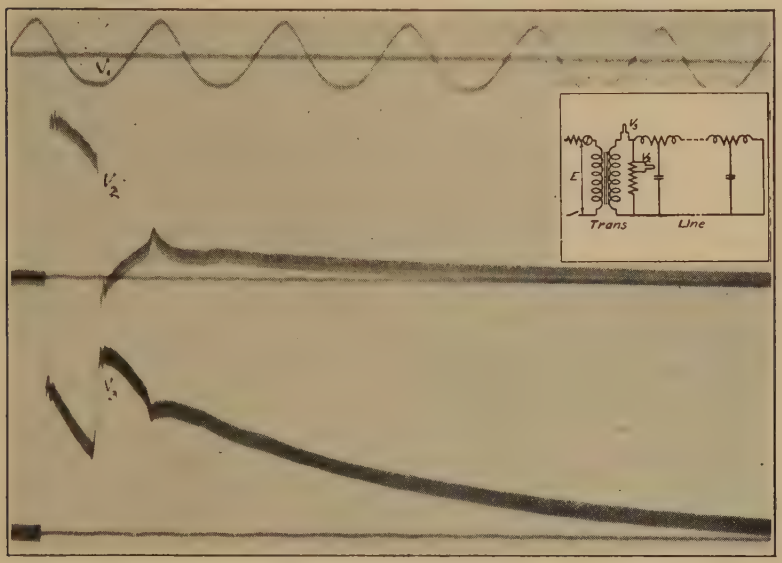


FIG. 130.—Traveling waves on artificial transmission line. Receiver end short circuited.

E = 100 volts; I = 12 amp.; L = 0.860 henry; C = 5.38 mf.; R = 100 ohms; v_1 = 100∼. Equivalent length of line, 400 miles; 4/0 copper.

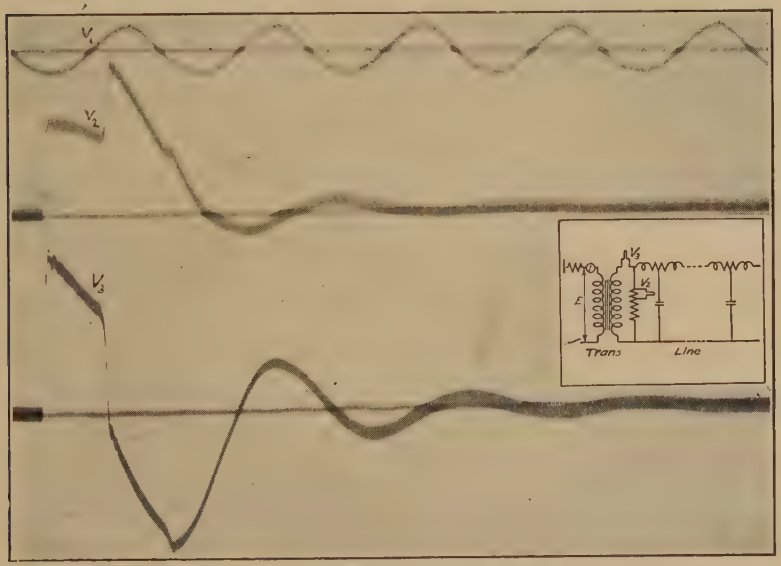


FIG. 131.—Traveling waves on artificial transmission line. Receiver end open.

E = 100 volts; I = 12 amp.; L = 0.860 henry; C = 5.38 mf.; R = 100 ohms; v_1 = 100∼. Equivalent length of line, 400 miles; 4/0 copper.

αt is the *time angle*. Under permanent or steady conditions with a simple sine voltage $^{M}E \sin \omega t$ impressed at the generating station, $\alpha = \omega$ and has only one value. However, if the impressed voltage is a complex wave or during transition periods between two permanent conditions while transient currents and voltages are flowing in the system, α would have more than one value.

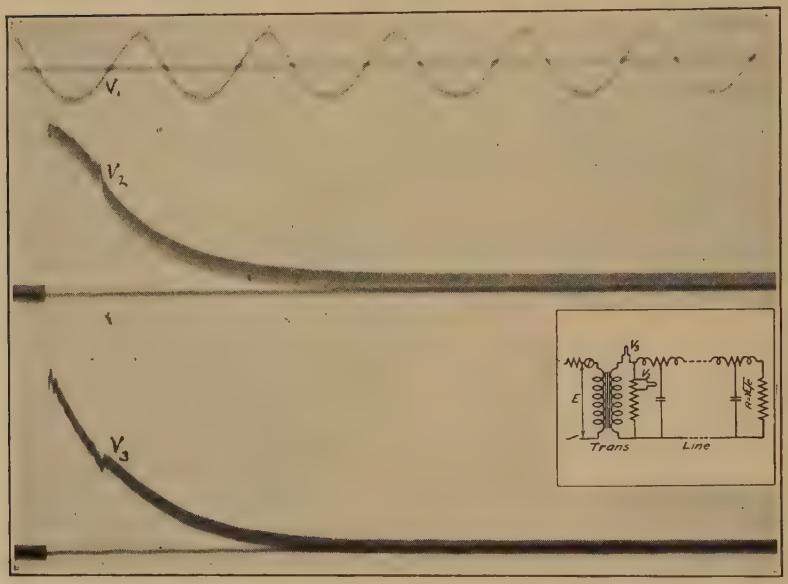


FIG. 132.—Traveling waves on artificial transmission line. Resistance at receiver end = $\sqrt{L}/\sqrt{C}$ ohms.

E = 100 volts; I = 12 amp.; L = 0.860 henry; C = 5.38 mf.; R = 100 ohms; $v_1 = 100\sim$. Equivalent length of line, 400 miles; 4/0 copper.

βx is the *space or distance angle* with x as the distance along the line from the origin. If waves of more than one frequency are passing over the line, β would have more than one value.

γ_1 and γ_2 are phase angles for $t = 0$.

Traveling Waves.—Traveling waves are in many respects similar to free oscillations or standing waves, as the transfer of energy between the dielectric and magnetic fields is the

basis for all double-energy electric phenomena. The essential difference is that in traveling waves power flows along the line while in free oscillations or standing waves the energy oscillates between the two fields but does not travel from one line element to another. Oscillograms of the current and voltage factors in traveling waves are shown in

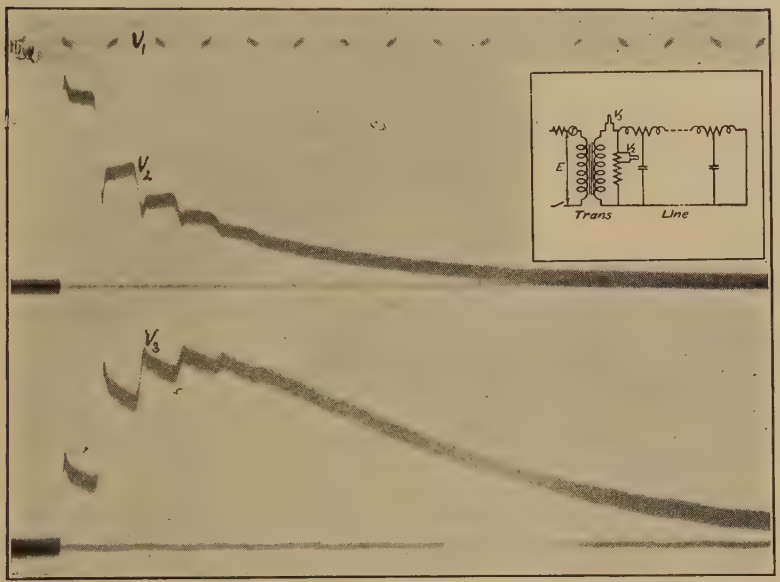


FIG. 133.—Traveling waves on a 200-mile artificial transmission line. Receiver end short circuited.

Line constants: $L = 0.41$ henry; $C = 2.75$ mf.; $R = 50.69$ ohms. R in voltage vibrator v_2 circuit = 2,200 ohms. R in current vibrator v_3 circuit = 2 ohms. I in primary = 24 amp. 3-kv.a. transformer; ratio, 5:1. $v_1 = 100\sim$.

Figs. 130 to 138. It should be noted that the current is in time phase with the voltage for the outgoing waves and differs by 180° for the returning waves. In both cases a flow of power occurs along the line.

In Fig. 133 the receiver end of the line is short circuited. The reflected voltage wave is in opposite time phase to the outgoing wave while the corresponding current waves are in the same direction.

In Fig. 134 the receiver end of the line is open and as a consequence the reflected current wave reverses in sign

while the corresponding voltage wave is in the same direction as the outgoing wave.

For the circuit in Fig. 135 a resistance equal to the surge impedance of the circuit $\sqrt{L}/\sqrt{C}$ is inserted at the receiver end of the line. All the energy of the traveling

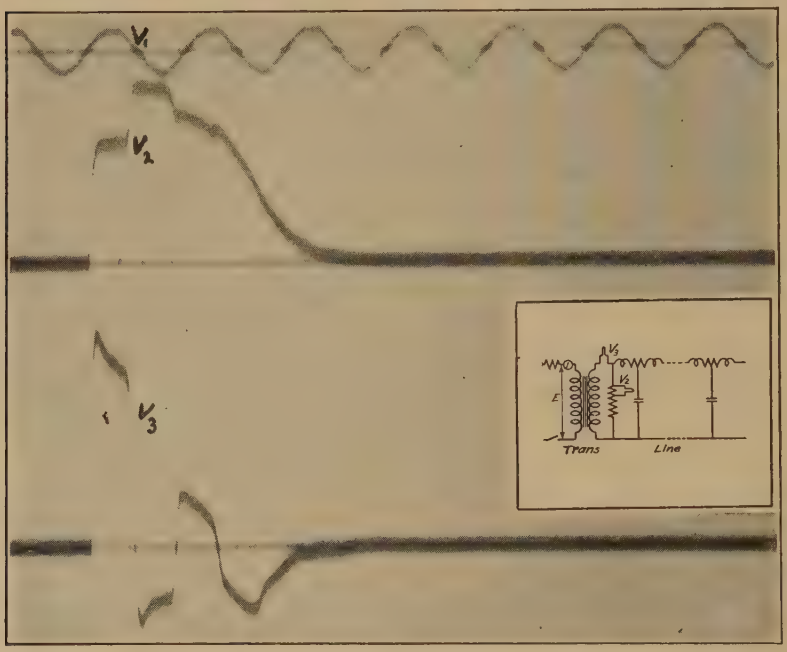


FIG. 134.—Traveling waves on 200-mile artificial transmission line. Receiver end open.

Line constants: L = 0.41 henry; C = 2.725 mf.; R = 50.69 ohms. R in voltage vibrator v_2 circuit = 4,000 ohms. R in current vibrator v_3 circuit = 2 ohms. E across primary = 4.6 volts; 3-kv.a. transformer; voltage ratio, 5:1; I in primary = 22.0 amp. v_1 = 200~.

wave was dissipated into heat by the Ri^2 losses at the receiver end of the line and as a consequence there was no reflected voltage or current waves or return flow of power. From the timing wave and known length of line it is found that the velocity of propagation of the impulse is approximately equal to v or $3 \cdot 10^{10}$ cm. per sec., the velocity of propagation of an electromagnetic field in free space.

A traveling wave in an electric line may be transformed into a standing wave, as illustrated by the oscillograms in Figs. 131 and 136. In Fig. 136, with the receiver end of the line open, both the voltage and current waves show that the traveling wave passed from the generator to the receiver end of the line and back again three times before

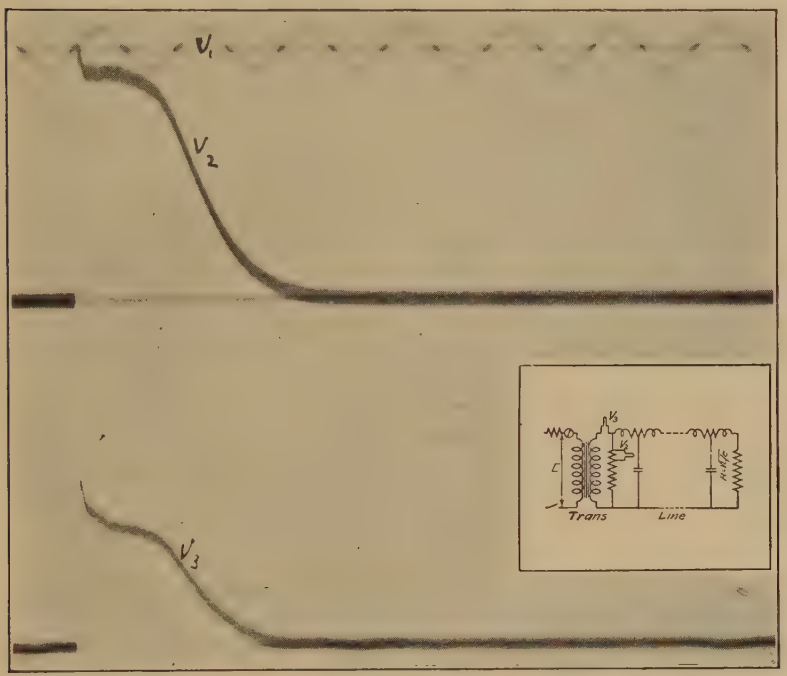


FIG. 135.—Traveling wave on a 200-mile artificial transmission line. Receiver

$$\text{end resistance} = \sqrt{\frac{L}{C}} = 388 \text{ ohms.}$$

Line constants: $L = 0.41$ henry; $C = 2.725$ mf.; $R = 50.69$ ohms. R in voltage vibrator v_2 circuit = 2,200 ohms. R in current vibrator v_3 circuit = 2 ohms. 3-kv.a. transformer; ratio, 5:1. $v_1 = 200\sim$.

it was changed into a standing wave. During this period the voltage and current waves are in phase or 180° apart, showing a flow of power along the line, but when the traveling wave is changed to an oscillation the current leads the voltage (note position of vibrators in the circuit diagram) by 90°. If the current leads or lags 90° with respect to

the voltage, the power in the circuit is reactive and therefore the average flow of power along the line is equal to zero.

Similarly, the oscillogram in Fig. 131 shows impulses which after passing over the lines twice as traveling waves are transformed into standing waves or oscillations. In each case the impulse starts as a traveling wave with the

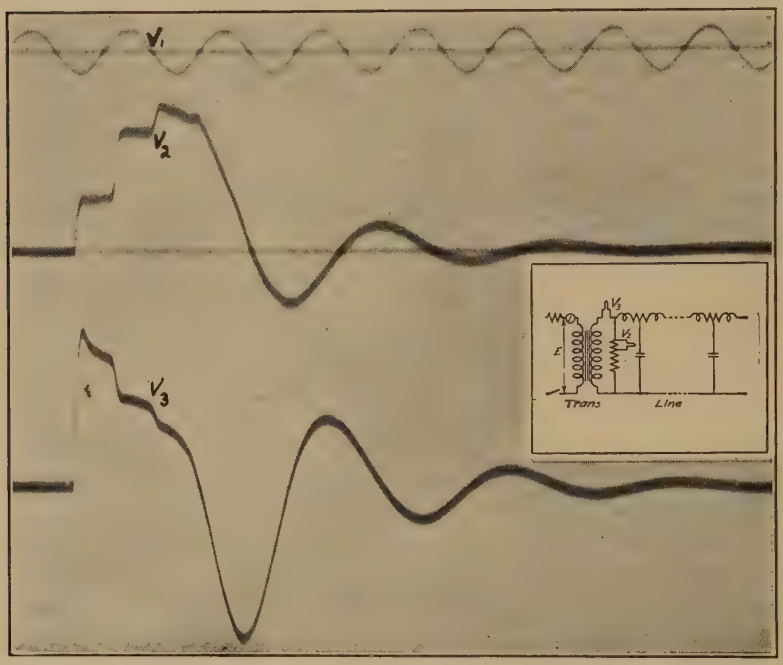


FIG. 136.—Traveling waves on 200-mile artificial transmission line. Receiver end open.

Line constants. L = 0.41 henry; C = 2.725 mf.; R = 50.69 ohms; R in voltage vibrator v_2 circuit = 2,240 ohms. R in current vibrator v_3 circuit = 2 ohms. 3-kv.a. transformers; ratio 10:1; I in primary = 6 amp. v_1 = 200~.

current and voltage in phase and a flow of power along the line. The oscillogram shows that the traveling wave was converted into an oscillation or standing wave, in which the current and voltage differ by 90° in time phase, in less than one-sixtieth of a second, and that the energy then oscillated between the magnetic and dielectric fields without flow of power along the line.

The change in frequency when the traveling wave is converted into a standing wave should be noted. In the traveling wave the inductance and condensance of the line alone determine the velocity of propagation while for the oscillations or standing waves the line and transformer may oscillate together as a compound circuit as in Figs. 142 and 143.

In determining the instantaneous values for the current and voltage at any point on the system the power flow must be taken into consideration in addition to the dissipation of the transient electric energy into heat as expressed by the damping factor ϵ^{-ut}. It is evident that the flow of power may be increasing, decreasing or unvarying in the direction of propagation.

If the power flow is uniform, the expressions for the current and voltage are in the simplest form, equations (250) and (251), as the power transfer factor does not appear in the equations.

$$i = I_0\epsilon^{-ut} \cos (\omega t \mp \phi x - \gamma) \tag{250}$$

$$e = E_0\epsilon^{-ut} \cos (\omega t \mp \phi x - \gamma) \tag{251}$$

$$p = E_0 I_0 \epsilon^{-2ut}[1 - \sin^2 (\omega t \mp \phi x - \gamma)] \tag{252}$$

$$\text{Average power } p = \frac{E_0 I_0}{2}\epsilon^{-2ut} \tag{253}$$

Uniform flow of transient power is infrequent but may occur in special cases. Thus if a transformer line and load, as in Fig. 141, are disconnected from the power supply and left to die down together, uniform flow of power in the line may be realized, provided the dissipation constant of the line is equal to the average dissipation constant of the whole system. Consider the transformer as having stored in the magnetic field a comparatively large quantity of energy, while its resistance and conductance are relatively small compared to the corresponding value for the line. Likewise assume that the load part of the circuit has very

little energy stored in its magnetic and dielectric fields and that its dissipation constant is large as compared to that of the line. Under these conditions the dissipation of energy is most rapid in the load part of the circuit and slowest in the transformer. Hence a flow of energy will occur from the transformer to the load. If the rate of energy dissipation of the line is midway between the corresponding rates for the load and transformers, the energy dissipated in the line would be equal to the amount initially stored in the line, while part of the energy originally stored in the transformer flows through the line and is dissipated in the load part of the circuit. The flow of power in the line would be uniform, as it delivers to the load part of the circuit all the energy received from the transformer.

The *flow of power decreases* along the line in the direction of propagation, if energy is left in the circuit elements as the traveling wave passes along the line. That is, the traveling wave scatters part of its energy along its path and thus decreases in intensity with the distance traveled. This decrease is expressed by a *power-transfer constant* s, comparable to the power-dissipation constant u. If no energy were supplied to the line by the traveling wave the voltage and current would decrease by the dissipation factor ϵ^{-ut}. With power supplied by the flow of energy the decrease would be slower and would be expressed by a combination of the damping and power-transfer factors.

For decreasing flow of power:

$$\text{Damping factor} = \epsilon^{-ut} \quad (254)$$

$$\text{Power-transfer factor} = \epsilon^{+st} \quad (255)$$

$$\text{Combined damping and power-transfer factor} = \epsilon^{-(u-s)t} \quad (256)$$

Similarly if the *flow of power increases* along the line in the direction of propagation the traveling wave receives energy from the line elements and the actual decrease in

the voltage and current is greater than indicated by the dissipation constant. The power transfer would in this case be negative, and the combined damping and power-transfer factor would be expressed by equation (259).

For increasing flow of power:

$$\text{Damping factor} = \epsilon^{-ut} \quad (257)$$

$$\text{Power-transfer factor} = \epsilon^{-st} \quad (258)$$

$$\text{Combined damping and power-transfer factor} = \epsilon^{-(u+s)t} \quad (259)$$

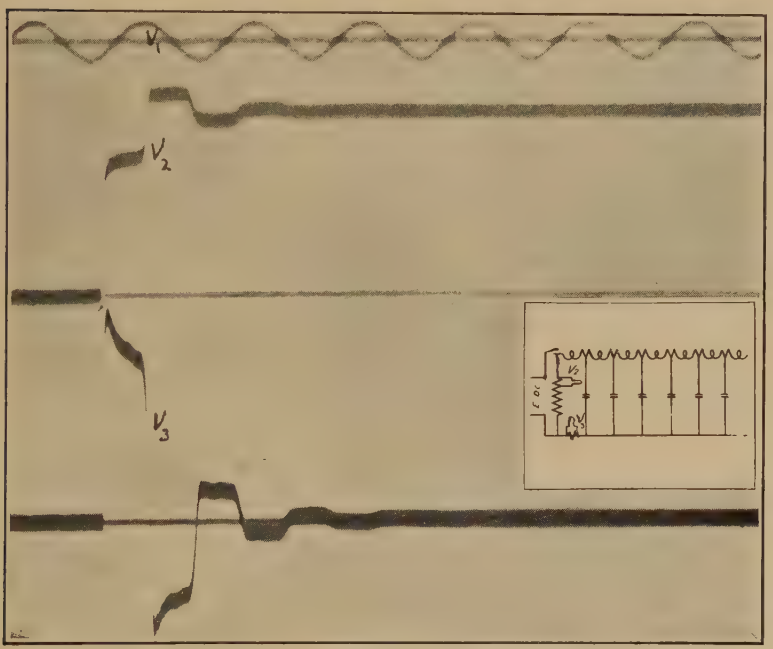


FIG. 137.—Traveling waves on a 200-mile artificial transmission line. Receiver end open. Direct-current voltage thrown on the line.

Line constants: $L = 0.41$ henry; $C = 2.725$ mf.; $R = 50.69$ ohms; $E = 240$ volts direct current. R in current vibrator $v_3 = 2$ ohms. $v_1 = 200\sim$.

To express the instantaneous values of the current and voltage at any point in the circuit a distance factor must be included, for if the traveling wave either scatters or gathers in energy as it travels along the line, the voltage and

current factors decrease at a lesser or greater rate, as the case may be, in the direction of propagation than if the flow of power were uniform. In order to use only one power-transfer constant s in the equation, let λ = the distance x expressed in velocity measure, equation (262).

For decreasing flow of power along the line:

$$\text{the distance damping factor} = \epsilon^{-s\lambda} \tag{260}$$

For increasing flow of power along the line:

$$\text{the distance damping factor} = \epsilon^{s\lambda} \tag{261}$$

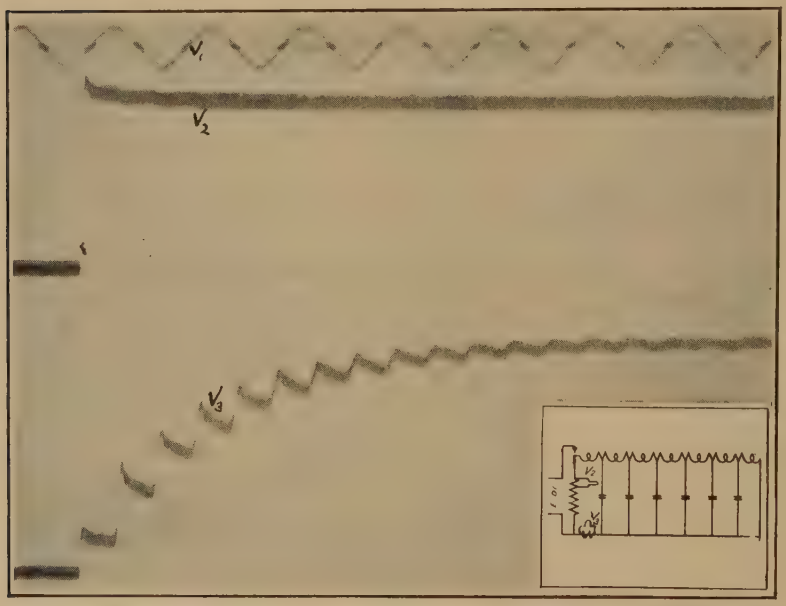


FIG. 138.—Traveling waves on a 200-mile artificial transmission line. Receiver end short circuited. Direct-current voltage thrown on the line.

Line constants: L = 0.41 henry; C = 2.275 mf.; R = 50.69 ohms; E = 240 volts direct current. R in current vibrator v_2 = 2 ohms; I = 4.4 amp. v_1 = 200~.

The instantaneous values of the transient current, voltage and power under conditions producing a flow of power along the line from the point of reference in the direction of propagation may be expressed by equations (262) and (263), or (264) and (265).

$$i = I_o \epsilon^{-(u \mp s)t \mp s\lambda} \cos(\omega t \mp \phi\lambda - \gamma) \quad (262)$$

$$e = E_o \epsilon^{-(u \mp s)t \mp s\lambda} \cos(\omega t \mp \phi\lambda - \gamma) \quad (263)$$

$$i = I_o \epsilon^{-ut \pm s(t-\lambda)} \cos(\omega t \mp \phi\lambda - \gamma) \quad (264)$$

$$e = E_o \epsilon^{-ut \pm s(t-\lambda)} \cos(\omega t \mp \phi\lambda - \gamma) \quad (265)$$

$$p = I_o E_o \epsilon^{-2ut \pm 2s(t-\lambda)} [1 - \sin^2(\omega t \mp \phi\lambda - \gamma)] \quad (266)$$

$$\text{Average power, } p = \frac{I_0 E_0}{2} \epsilon^{-2ut \pm 2s(t-\lambda)} \quad (267)$$

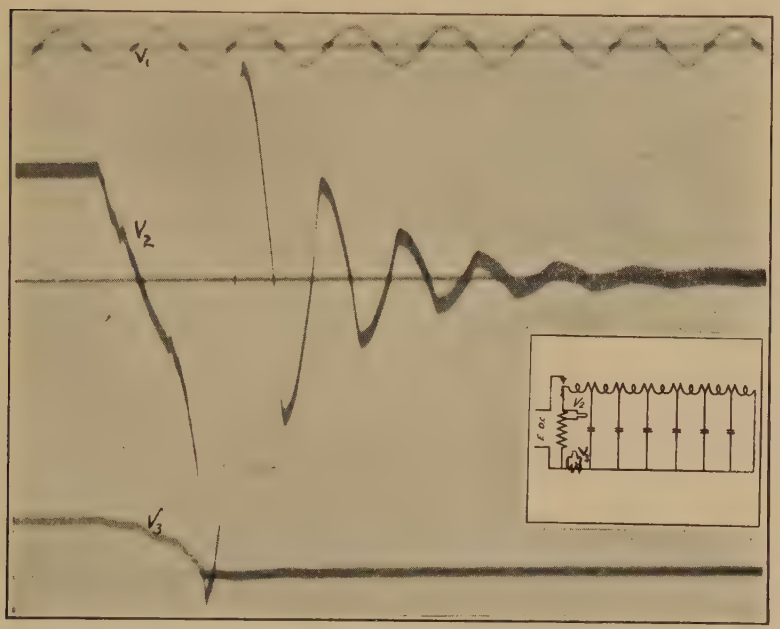


FIG. 139.—Oscillations of a 200-mile artificial transmission line. Receiver end short circuited. Switching off direct-current voltage.

Line constants: $L = 0.41$ henry; $C = 2.725$ mf.; $R = 50.69$ ohms. $E = 124$ volts direct current; $I = 2.5$ amp.; v $= 200\sim$.

The upper sign of $\phi\lambda$ applies to waves traveling in the direction of increasing values of λ and the lower sign for returning waves, for which λ is decreasing. For $s = 0$, which represents a constant flow of power, equations

(264) and (265) become identical with equations (250) and (251). Referring to Fig. 141, already used for illustrating the flow of constant power, it is evident that if the dissipation constant for the line is less than the average dissipation constant for the system the flow of power from the transformer will be such as to increase the power stored in the line, while if the line dissipation constant is greater than the average the reverse would be the case.

Traveling waves are of very frequent occurrence in electric power systems. Not merely such violent disturbances

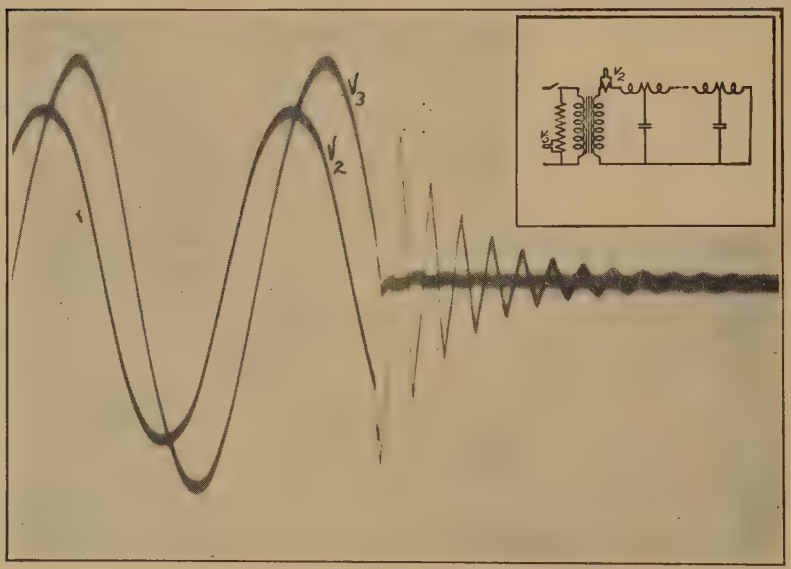


FIG. 140.—Switching off a 200-mile artificial transmission line. Receiver end short circuited.

Line constants: $L = 0.41$ henry; $C = 2.725$ mf.; $R = 50.69$ ohms; 3-kv.a. transformer; ratio, 5:1; $I = 6$ amp. in primary of transformer lags behind the voltage. $f = 60\sim$.

as direct strokes of lightning or short circuits, but practically every change in load or circuit conditions produces transient waves that travel over the system. Simple traveling waves as illustrated by the oscillograms in Figs. 133 to

135 are frequently called *impulses*. In the first stage of the impulse, as it passes along a line, the wave energy increases at a rate depending on the steepness of the wave front, and after the maximum value is reached the wave energy decreases. While the wave energy increases the combined dissipation and power-transfer factor is represented by $\epsilon^{-(u+s)t}$ as in equation (259), and during the decreasing stage by $\epsilon^{-(u-s)t}$ as in equation (256). The steepness of the wave front which corresponds to the sharpness or suddenness of a blow is often a more important factor in causing damage to the electric system than the quantity of energy involved.

Compound Circuits.—In commercial systems the transmission line is not an independent unit but merely a link between the generator and load circuits. Step-up and

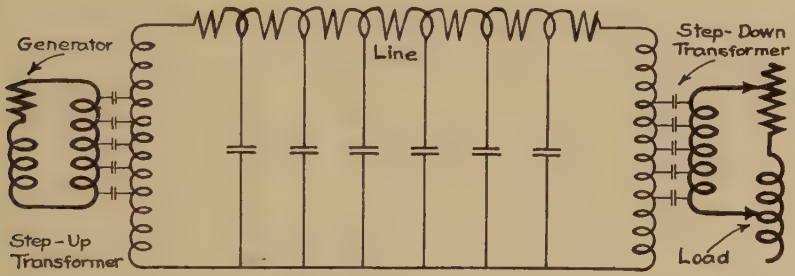


FIG. 141.—Circuit diagram of a compound circuit.

step-down transformers, generators and load circuits, lightning arresters and regulating devices, and all the apparatus necessary for the operation of the system are electrically interconnected into one unit. In the several parts of the system the circuit constants differ in relative magnitude and hence the velocity of propagation of an electric impulse varies and no two sections may have the same natural period of oscillation. While the whole system may oscillate as a unit, partial oscillations are of much more frequent occurrence.

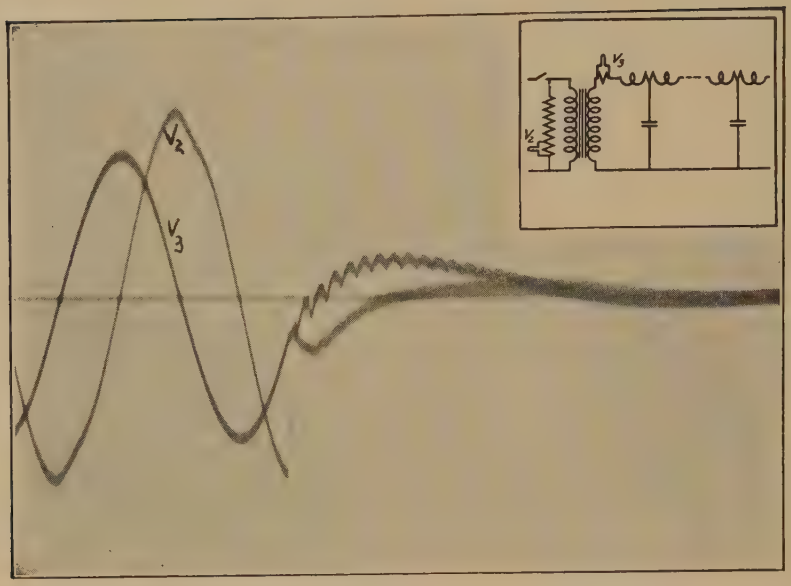


FIG. 142.—Oscillations of a compound circuit. A 3-kv.a. transformer; ratio, 5:1, and a 200-mile artificial transmission line. Receiver end open.

Line constants: $L = 0.41$ henry; $C = 2.725$ mf.; $R = 50.69$ ohms. $E = 100$ volts alternating current; $I = 1$ amp. in primary; $f = 60\sim$.

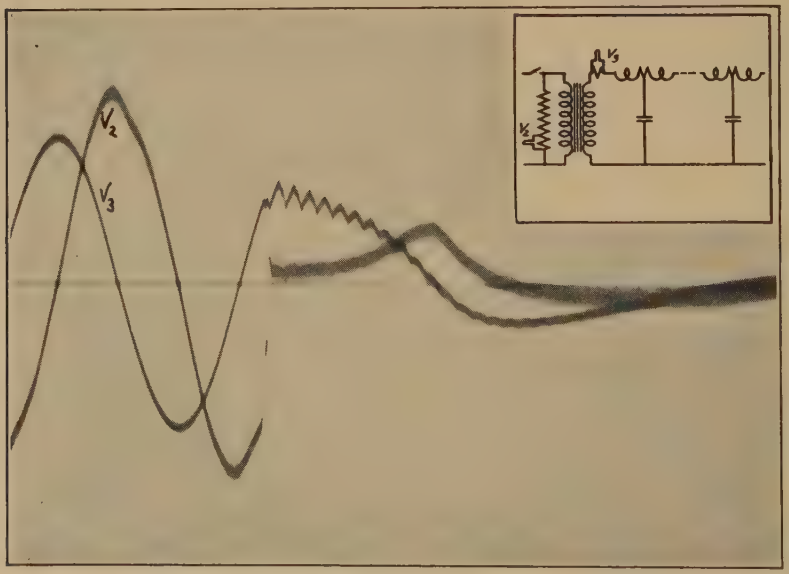


FIG. 143.—Oscillations of a compound circuit. A 3-kv.a. transformer, ratio, 5:1, and a 200-mile artificial transmission line; receiver end open.

Line constants: $L = 0.41$ henry; $C = 2.725$ mf.; $R = 50.69$ ohms; $E = 100$ volts; $I = 1$ amp. in primary; current leads the voltage. $f = 60\sim$.

Problems and Experiments

1. Given a transmission line, 80 miles long, of No. 0000 copper, spaced 12 ft. and with the receiver end open. From handbook tables obtain the line constants. Find the fundamental oscillation frequency of the line. Check the results by solving for the frequency from the known velocity of propagation of an electric wave in space and use the given length of the line.

2. Make a series of oscillograms similar to Figs. 126 to 132, on an artificial transmission line. From the oscillograms determine the equivalent length of actual line. Check by determining the natural frequency of oscillation from the line constants.

3. Make oscillograms similar to Figs. 139 and 140.

4. In the oscillogram in Fig. 142 the ripples on the current wave indicate reflections of traveling waves in the transmission line with the receiver end open. Calculate the length of the line.

5. From the oscillograms and data in Fig. 142 calculate the inductance in the transformer in the compound circuit. Assume the condensance of the transformer equal to zero. It should be noted that the inductance is essentially massed while the condensance is distributed, and hence for the combined circuit $f = \frac{1}{2\sqrt{2\pi LC}}$.

6. From the oscillogram and data in Fig. 143 make calculations similar to those asked for in Problems 4 and 5 for Fig. 142.

7. Make oscillograms of the oscillations of compound circuits, similar to Figs. 142 and 143.

CHAPTER VII

VARIABLE CIRCUIT CONSTANTS

In the preceding chapters the fundamental laws of transient electric phenomena are derived under the assumption that in any given circuit the resistance, inductance, conductance and condensance, the so-called *circuit constants*, remain constant in value during the transition period under discussion. The transients are due to changes in circuit condition or in the impressed voltage, but during the period required for the dissipation of the stored energy or the readjustment of the energy content in the system the values of R, L, G and C are assumed constant. The oscillograms, Chaps. III to VI inclusive, of electric transients were obtained from circuits in which the resistance, inductance, conductance and condensance remained essentially constant.

It is evident that if the circuit constants do not remain constant during the period the transients occur, but vary rapidly over a wide range of values, the nature of the resulting electric phenomena must be correspondingly more complex. The laws for the variations in R, L, G and C are not always known or are so complex that they can not be represented in the form of equations. For example, data for the quantitative ratios between the magnetomotive force and the resulting magnetic flux in iron-clad circuits, as indicated by the hysteresis loop, may readily be obtained experimentally, but it has not been possible to express the general relation in the form of an exact mathematical equation. The empirical equations in common use are limited in their application and give only approximate values.

Variable Resistance.—Change in temperature is the most important factor in producing variations in the resistance of electrical conductors, *the R circuit constant.* For

metals the specific resistance is a linear function of the temperature over a fairly wide range.

$$\rho_t = \rho_0 + \alpha t^\circ \tag{268}$$

ρ_t = specific resistance at t°C.
ρ_0 = specific resistance at 0°C.
α = temperature coefficient.

For rapid changes in temperature the rate of change in the resistance may be large, as illustrated by the oscillograms in Figs. 144, 145. For the tungsten incandescent lamp, Fig. 144, a starting transient appears in the current due to a rapid increase in the resistance of the filament as the temperature rises. When the switch is closed the filament is at room temperature and the resistance low. The current flowing through the lamp rapidly heats the filament to incandescence with an accompanying increase in the resistance and a decrease in the current. The timing wave shows that it required about 0.05 sec. for the lamp to reach approximately full brilliancy. During this period the resistance of the filament increased by over 500 per cent of its initial value.

For carbon the resistance decreases with an increase in temperature, or the temperature coefficient is negative, as illustrated in Fig. 145, showing that the time required for the resistance to reach a constant value was approximately 0.4 sec. and that the resistance of the incandescent carbon filament is about 50 per cent of its value at room temperature.

The temperature of the lamp filament increases until the dissipation of heat by radiation from the lamp is equal to the heat generated by Ri^2 losses. For a direct-current supply with constant impressed voltage the constant temperature condition is quickly reached. For alternating currents the power supplied to the lamps pulsates with double the current frequency and as the lamp emits or radiates heat continuously the temperature, and therefore the resistance of the filament, pulsates. This is illustrated

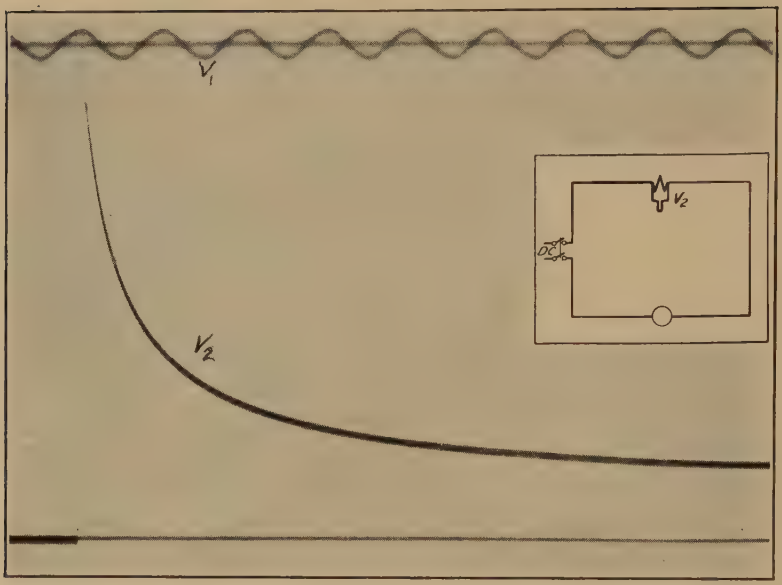


FIG. 144.—Starting transient of a 50-watt, 120-volt tungsten incandescent lamp.
E = 99 volts; v_1 = 100∼.

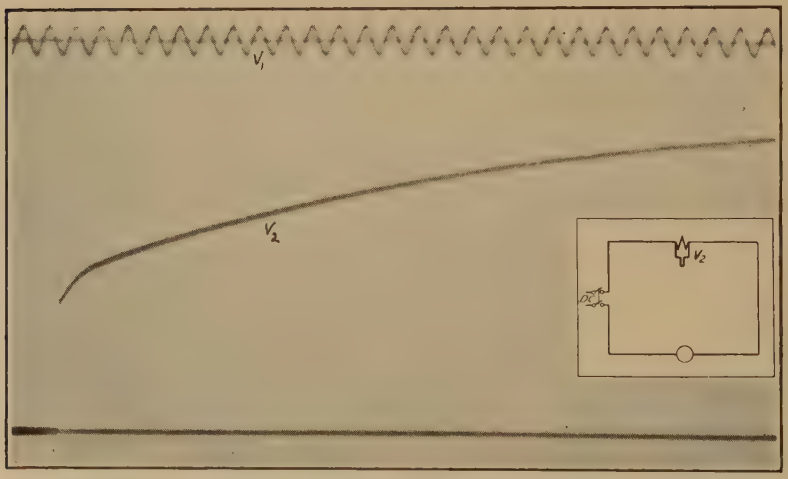


FIG. 145.—Starting transient of a 60-watt, 120-volt carbon lamp.
E = 98 volts; v_1 = 100∼.

by the oscillogram in Fig. 146. Alternating currents are impressed on two pairs of tungsten and carbon lamps, arranged as shown in the circuit diagram, with the vibrator of the oscillograph in the bridge connection. Since the resistance of the tungsten lamp increases and the carbon lamp decreases with an increase in temperature, the pulsations in the Ri^2 losses unbalance the bridge, as indicated

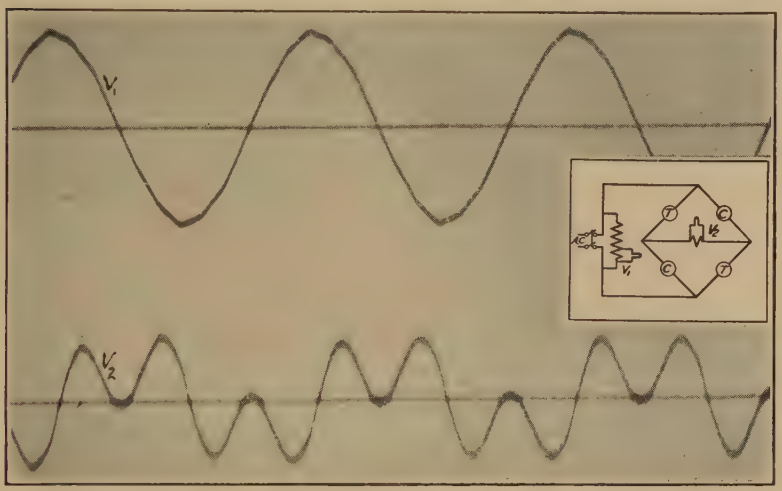


FIG. 146.—Pulsating resistance in tungsten T and carbon C lamps. $v_1 = 60\sim$.

by the pulsations in the currents flowing through the vibrator.

The resistance of the electric arc depends on many factors and may vary over a wide range with extreme rapidity. Since the resistance of the arc decreases with the increase in temperature, the arc alone is unstable and hence must be provided with a "ballast" to make continuous operation possible. On alternating currents an inductance placed in series with the arc serves as the stabilizer and the variations in the resistance of the arc are counterbalanced by the induced voltage in the inductance. In direct-current arc lamps a series resistance serves the same purpose.

In commercial systems the electric arcs that affect the series resistance, *the R circuit constant*, occur chiefly in the opening of switches. In breaking the circuit under load, especially when a large quantity of energy is stored magnetically in the circuit, arcs form in which the resistance varies rapidly from zero at start to infinity when the circuit is open.

This is illustrated by the oscillogram in Fig. 51. In the opening of the switch an arc forms whose resistance rapidly increases, approaching infinity when the circuit opens, which occurs at the point of maximum value in the voltage curve. The increase in the resistance can be determined quantitatively from the oscillogram by combining data from the rapidly increasing voltage and decreasing current curves.

Variable Inductance.—In iron-clad circuits as in transformers the magnetic flux is not directly proportional to the ampere-turns or magnetizing force. Hence the inductance, *the L circuit constant*, is not constant but varies with the permeability of the iron. Moreover, the variation in the inductance is different for decreasing and increasing flux values and depends on the maximum flux density as indicated by the form of the hysteresis loop. As no satisfactory mathematical expression has yet been found for the hysteresis cycle, solutions of practical problems are obtained by a series of approximations. As a first step in obtaining the shape of transients in iron-clad circuits, neglecting the difference between increasing and decreasing flux values, Fröhlich's formula is generally used.

$$\frac{H}{B} = v + \sigma H \tag{269}$$

The formula is based on the assumption that the permeability of the iron is proportional to its remaining magnetizability and states that the reluctivity of an iron-clad circuit is a linear function of the field intensity.

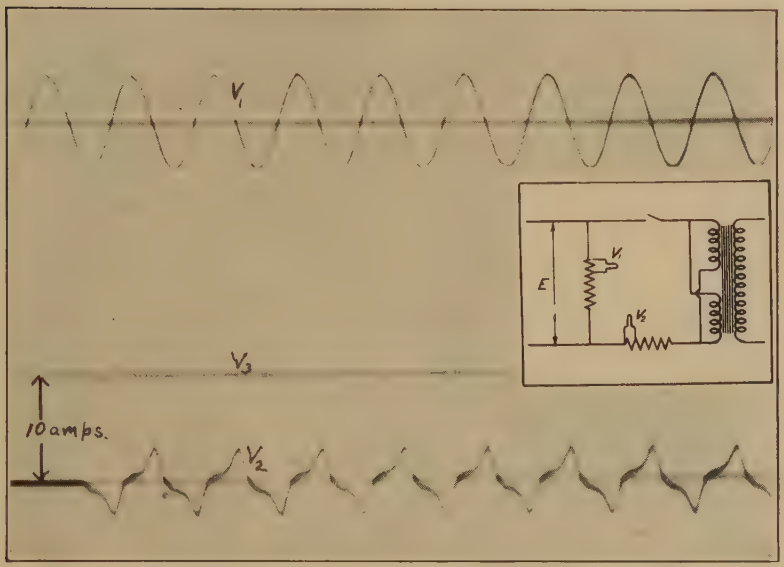


FIG. 147.—Transformer exciting current. No starting transient.

10-kv.a., 110,220/2,200 volts, 60-cycle. E = 106 volts; v_2 = exciting current; v_3, calibration current = 10 amp.

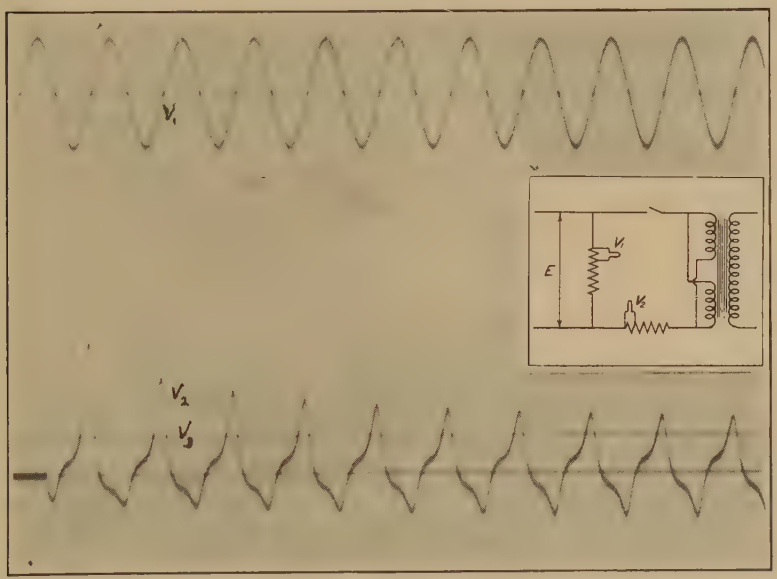


FIG. 148.—Starting transient of exciting current in a 3-kv.a., type-H, 60-cycle, 110,220/2,200 distribution transformer.

E = 116 volts; primary current = 0.73 amp. Direct-current calibration current, v_3 = 1.0 amp.

The effect of variable inductance in iron-clad circuits may be illustrated by the starting transients of alternating-current transformers. The magnitude of the starting-current transient depends more on conditions affecting the value of the inductance in the circuit than on what point on the voltage cycle the switch is closed. The direction and magnitude of the residual magnetism are important

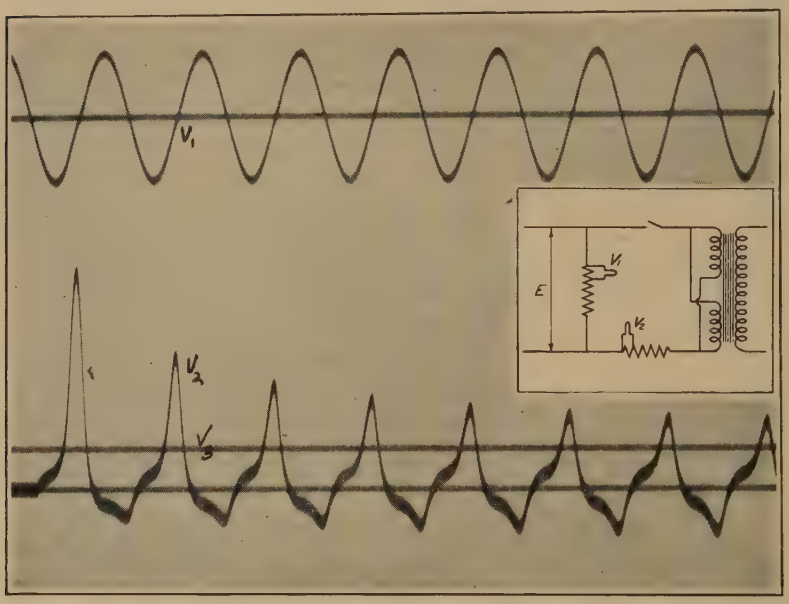


FIG. 149.—Starting transient of exciting current in a 3-kv.a., type-H, 60-cycle, 110,220/2,200 distribution transformer.

E = 116 volts; primary current = 0.73 amp. Direct-current exciting current, v_3 = 1.0 amp.

factors, as a combination of much residual flux with an additional magnetizing force in the same direction may bring the flux density in the core beyond the saturation point and hence greatly reduce the inductance in the circuit.

For the oscillograms in Figs. 147 to 151 a constant alternating-current voltage of sine wave shape was impressed on the transformer terminals. In Fig. 147 the switch was closed at an instant the magnetizing current would have been zero, if the circuit had been closed earlier, and hence no

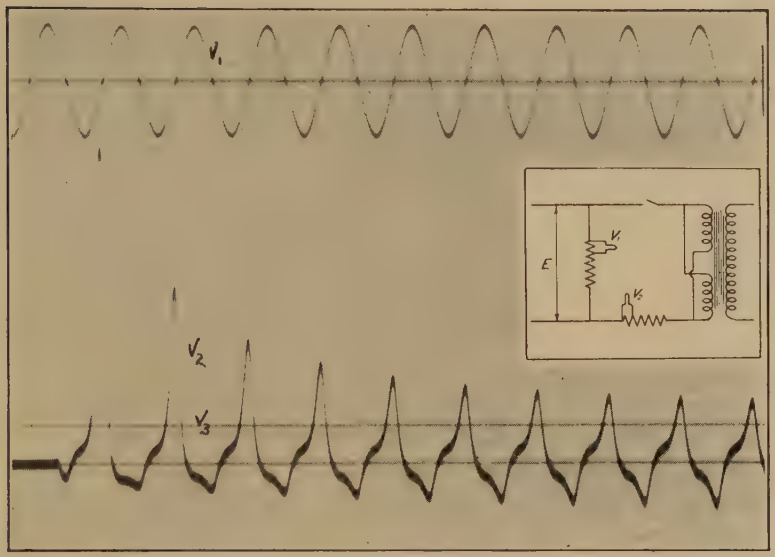


FIG. 150.—Starting transient of exciting current in a 3-kv.a., type-H, 60-cycle 110,220/2,200-volt transformer.

E = 116 volts; primary current = 0.73 amp. Direct-current calibration current, v_3 = 1.0 amp.

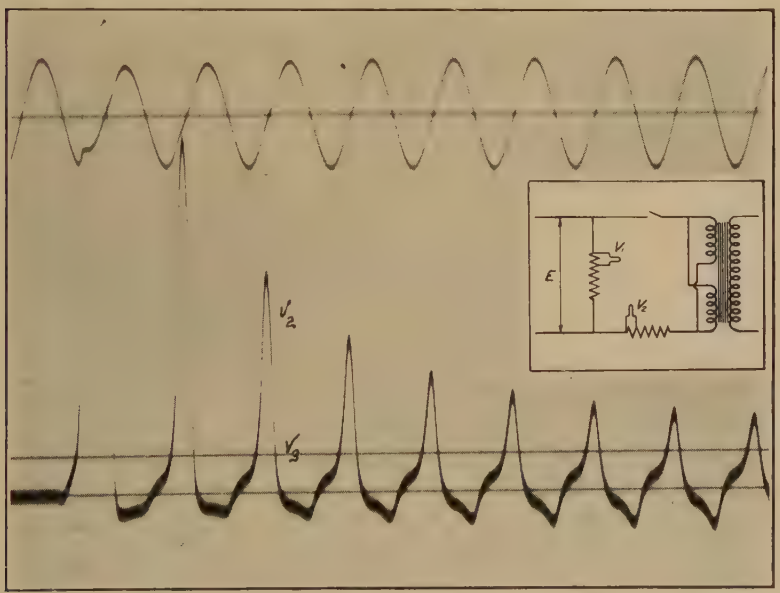


FIG. 151.—Starting transient of exciting current in a 3-kv.a., type-H, 60-cycle, 110,220/2,200-volt distribution transformer.

E = 116 volts; primary current = 0.73 amp. Direct-current calibration current, v_3 = 1.0 amp.

starting transient. In Figs. 148 to 151 the switch was thrown at other than the zero point of the magnetizing current cycle. The impressed voltage was only slightly above normal and the increase in the exciting current would not be large unless the iron core had residual magnetism in phase with the first half cycle.

The starting transient in Fig. 151 differs greatly both in form and magnitude, as compared to Fig. 149, although the impressed voltage was the same and the circuits were closed in the two cases at approximately the same point on the voltage wave. Apparently, the residual magnetism in the iron core, Fig. 151, was in the same direction as the flux produced by the magnetizing current during the first half cycle. Above saturation of the iron core the permeability is low and hence the transformer inductance is relatively small and therefore the first half cycle shows a correspondingly large current transient. A smooth curve drawn through the successive maxima values of the starting transient in Figs. 148, 149 or 150 could, with a fair degree of accuracy, be expressed by the exponential equation; but the corresponding curve drawn through the successive maxima values of the current-time curve in Fig. 151 would at the start have a much steeper initial gradient due to the variation in the inductance L of the transformer winding.

The same effect, due to variable inductance, may be obtained in breaking the field circuit of a direct-current generator, as illustrated by the oscillogram in Fig. 51. The change in the voltage and current curves from the instant the jaws of the switch separate to the peak value of the voltage is largely due to a change in the arc resistance. After the arc breaks, at the peak of the voltage curve, the vibrator circuit provides a path for the dissipation of the energy stored in the field. This voltage-time curve can not be expressed by the exponential equation because the inductance changes in value, as the permeability of the iron varies during the discharge. As the resistance in the

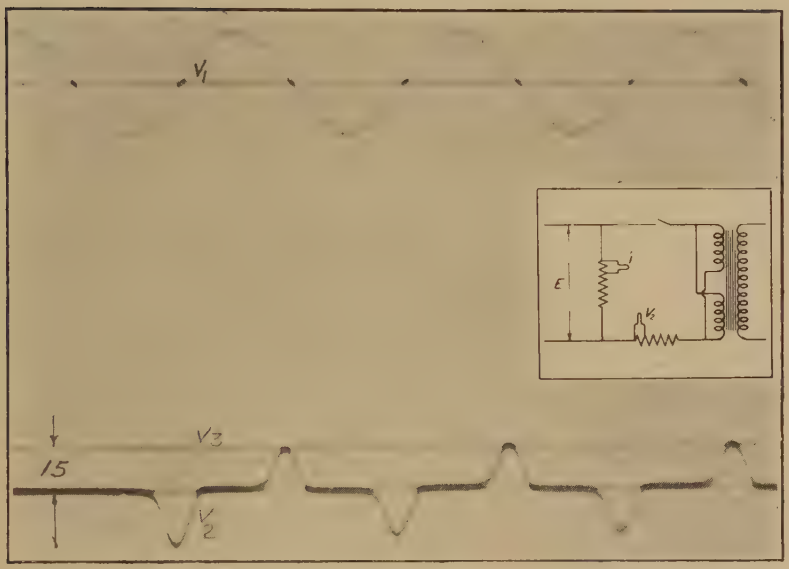


FIG. 152*a*.—Exciting current of overexcited, 3-kv.a., 60-cycle, 110/2,200-volt transformer. No starting transient.

E = 158 volts; v_2 = exciting current; v_3 = direct-current calibration current = 15 amp.

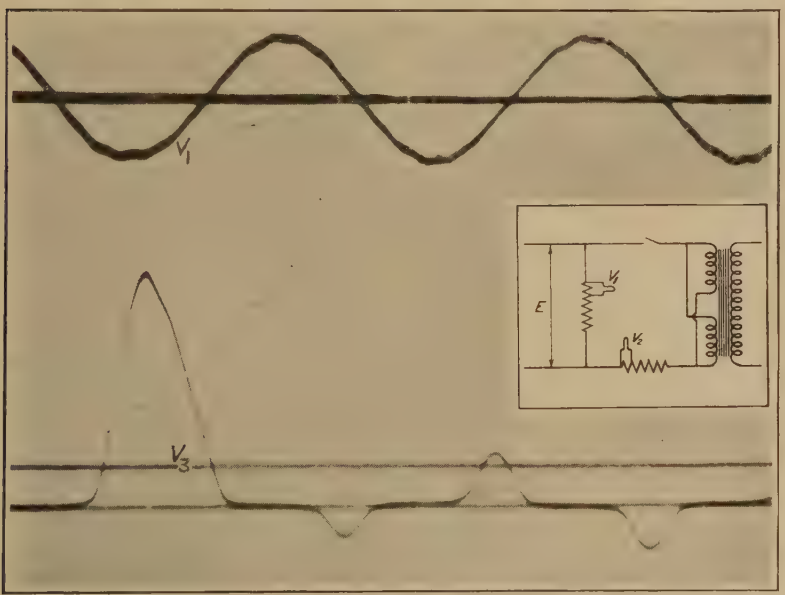


FIG. 152*b*.—Starting transient of overexcited, 3-kv.a., 115-volt transformer. E = 160 volts, I (calibration) = 10 amp.; v_1 = 60~.

vibrator circuit is constant, the voltage curve also represents the transient current.

The corresponding variation in the inductance when the generator field is formed is evidenced by the starting field current and armature voltage curves shown in Fig. 152.

Variable Conductance.—In the calculations on power transmission lines and, in general, for constant-potential systems in good condition, the leakage through the insula-

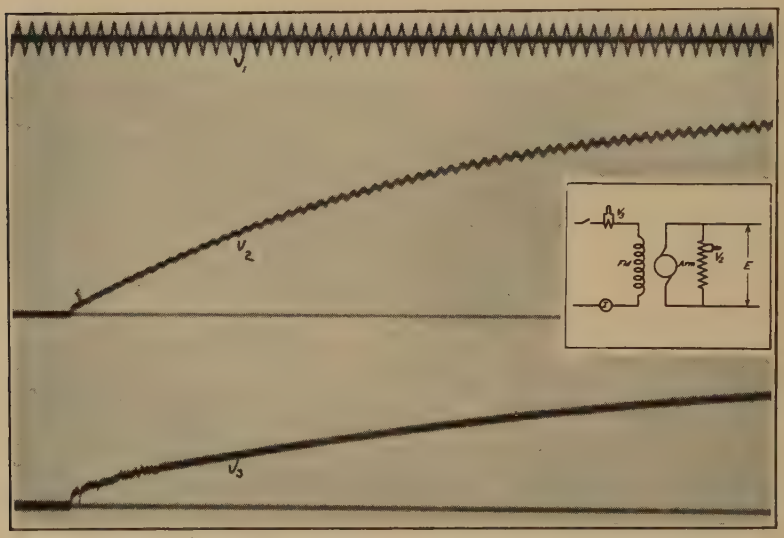


FIG. 153.—Building up the field of a 220-volt, 7½-kw. generator.
Armature voltage = 167 volts; field current = 2.9 amp.; speed 1,800 r.p.m., $v_1 = 100\sim$.

tion is small, so that the conductance is negligible and *the G circuit constant* may be taken as equal to zero. The insulation of electric circuits deteriorates with varying rates and the conductance and leakage increase and may become very large, as, for example, if the insulation completely breaks down and a short circuit is formed. A rupture of the insulation or any sudden change in the conductance of the electric circuit will of necessity cause violent disturbances in the system. Arcing grounds or intermittent arcs,

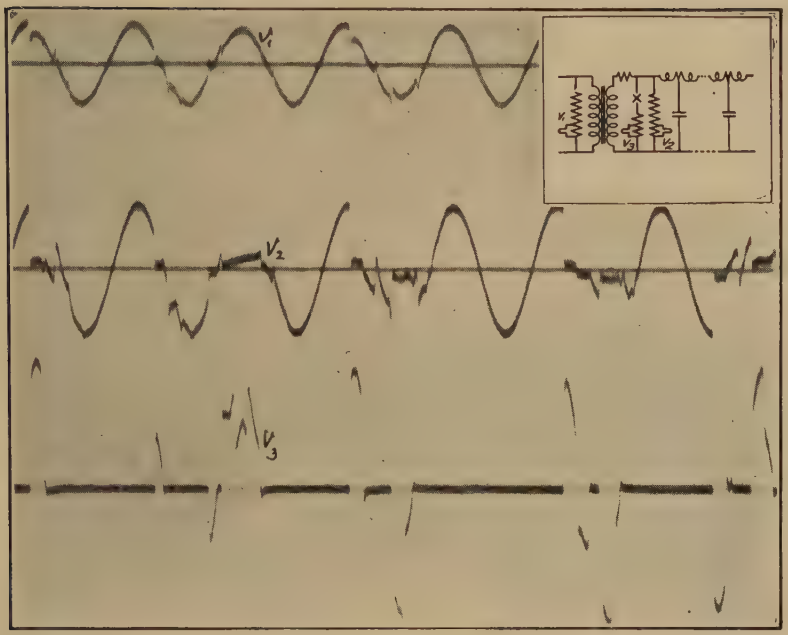


FIG. 154.—Intermittent arcing near generator end of 200-mile artificial transmission line.
Copper electrodes. Supply voltage, 100 volts. Transformer ratio, 1:5; $f = 60\sim$.

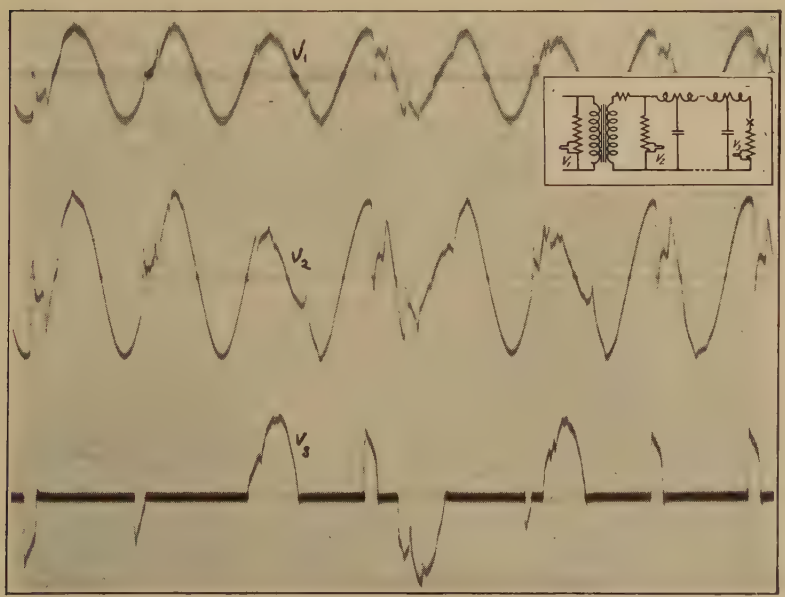


FIG. 155.—Intermittent arcing at receiver end of 200-mile artificial transmission line.
Copper electrodes. Supply voltage 85 volts. Transformer ratio, 1:5; $f = 60\sim$.

as illustrated by the oscillograms in Figs. 153 to 156, are prolific sources of electric transients. It is evident that momentary short circuits, as would be produced by an intermittent arcing ground with the conductance varying practically from zero to infinity at an extremely rapid rate, would give rise to oscillations of any frequency and produce waves and impulses that would travel to all parts of the system.

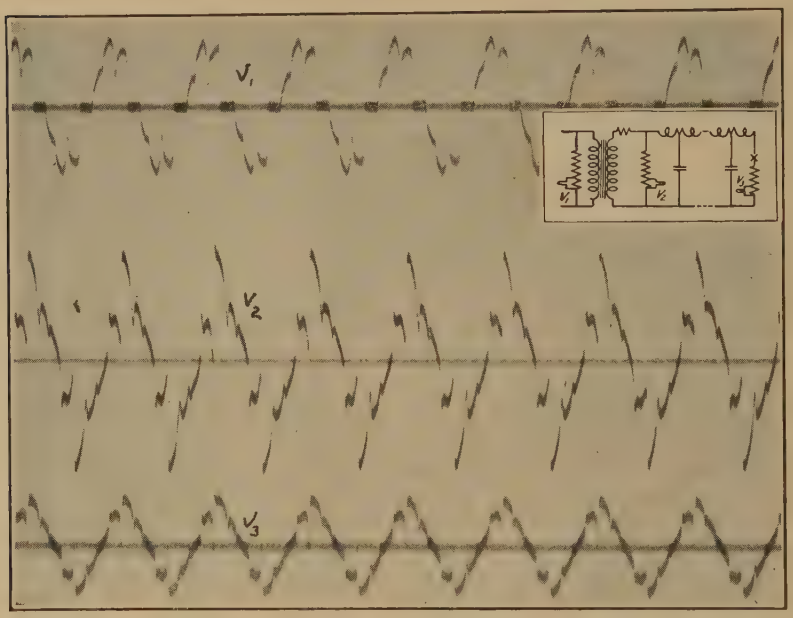


FIG. 156.—Continuous arc at receiver end of 200-mile artificial transmission line. Copper electrodes. Primary voltage on transformer, 90 volts alternating current; transformer ratio, 1:5; $f = 60\sim$.

Variable Condensance.—Under ordinary conditions and for low voltages, air is very nearly a perfect insulator. In other words, the conductivity of air is practically zero, the permittivity unity and the energy loss extremely small. If the voltage is increased until the limit of the insulating strength of the air is reached, important changes occur in both the electric and dielectric circuit constants.

It has been found experimentally that corona appears on wires of different sizes for a constant voltage gradient, at a distance of $0.301\sqrt{r}$ cm. from the surface of the conductor, r being the radius in centimeters of the wire. To produce corona under normal conditions (25°C. temperature and 76-cm. pressure) requires a voltage gradient of at least 29.8 kv. per cm. For air at other than normal temperature and pressure conditions, a density factor δ,

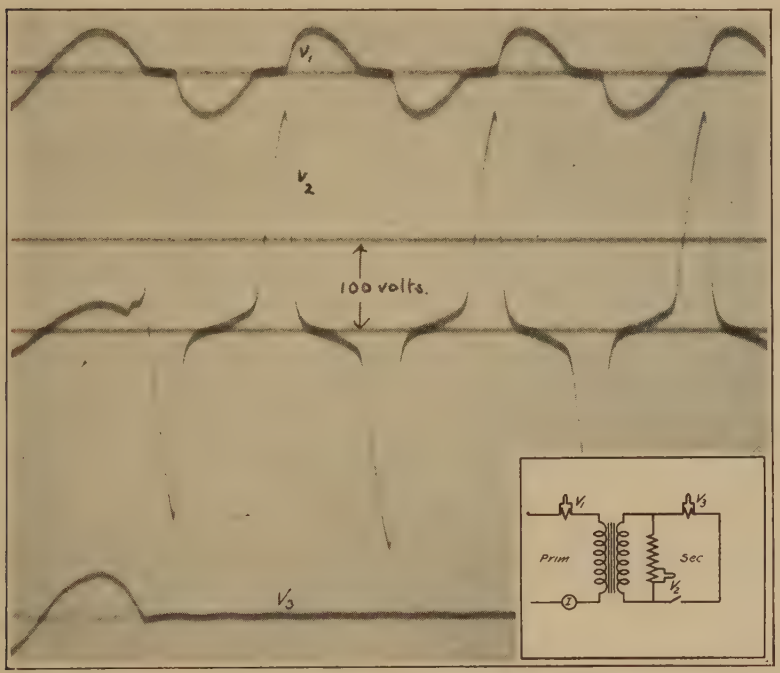


FIG. 157.—Opening of secondary circuit on a current transformer.
Primary current, 23.5 amp.; ratio, 5:1; v_1 = 60~; resistance in vibrator v_2 circuit = 3,200 ohms; calibration voltage = 100 volts direct-current.

as given in equation (270), must be introduced in the equations.

$$\delta = \frac{3.92b}{273 + t} \tag{270}$$

b = barometric pressure in centimeters.
t = temperature in Centigrade degrees.

For parallel wires, as in power-transmission lines, the surface gradient and voltage to neutral required to produce visual corona are given in equations (271) to (274).

$_vG$ = visual corona voltage gradient at surface of conductor.

$_ve$ = visual corona voltage.

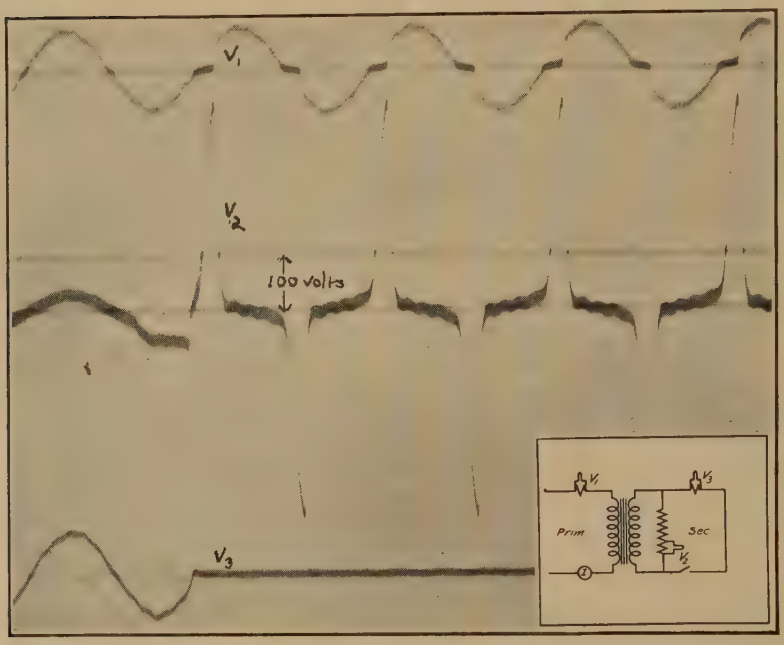


FIG. 158.—Opening of secondary circuit on a current transformer.

Primary current, 24 amp.; ratio, 5:1; v_1 = 60~; resistance in v_2 circuit = 6,000 ohms; calibration voltage = 100 volts direct-current.

$_vE$ = effective value of sine wave visual corona voltage.

^m_vE = maximum of $_vE$, sine wave.

r = radius of conductor in centimeters.

d = distance between transmission-line conductors in centimeters.

$$_vG = 29.8\, m\delta \left(1 + \frac{0.301}{\sqrt{\delta r}}\right) \text{kv. per cm.} \qquad (271)$$

$$_{v}e = 29.8\, m\delta \left(1 + \frac{0.301}{\sqrt{\delta r}}\right) r \log_{\epsilon} \frac{d}{r} \text{ kv. to neutral} \quad (272)$$

$$^{M}_{v}E = 29.8\, m\delta \left(1 + \frac{0.301}{\sqrt{\delta r}}\right) r \log_{\epsilon} \frac{d}{r} \text{ kv. to neutral} \quad (273)$$

$$_{v}E = 21.1\, m\delta \left(1 + \frac{0.301}{\sqrt{\delta r}}\right) r \log_{\epsilon} \frac{d}{r} \text{ kv. to neutral} \quad (274)$$

With the occurrence of visual corona in high-voltage circuits the conductivity of the air in the space filled by

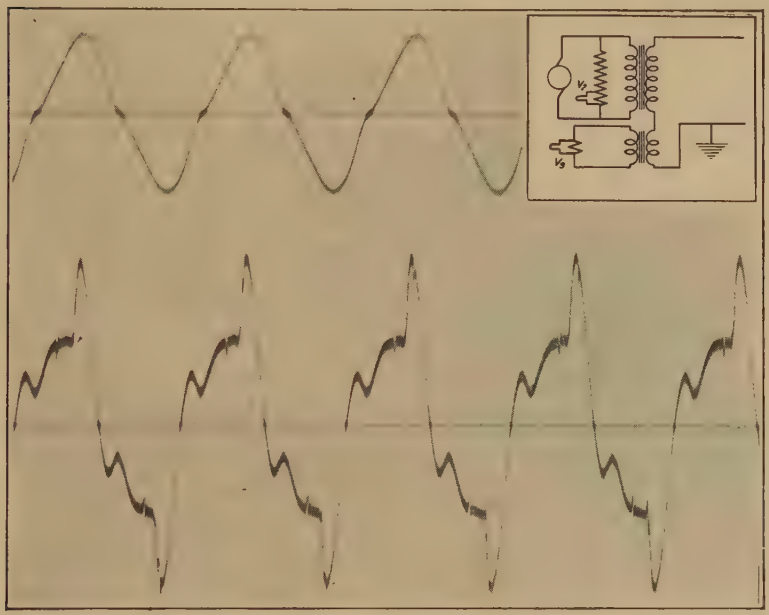


Fig. 159.—Corona. Variable condensance.
E = 33 kv.; length of line, 88.4 ft.; spacing, 4 in.; brass wire, No. 24 A.w.g.

the corona is increased. Thus in circuits with parallel wires as high-tension transmission lines a voltage gradient above 29.8 kv. per cm. will produce corona in the air surrounding the conductor surface and this space filled by the corona glow becomes semi-conducting. This produces a change in the circuit condensance as with the appearance of the corona the effective size of the conductor, and hence

of the condenser surface, is increased. The apparent increase in the condensance with the appearance of corona may to a large extent be due to the harmonics introduced by the pulsating nature of the corona. This would increase the condensance current and give indications on the ammeter similar to what would result from an increase in the condensance of the circuit. For alternating currents the visual corona is intermittent, appearing only near the peaks

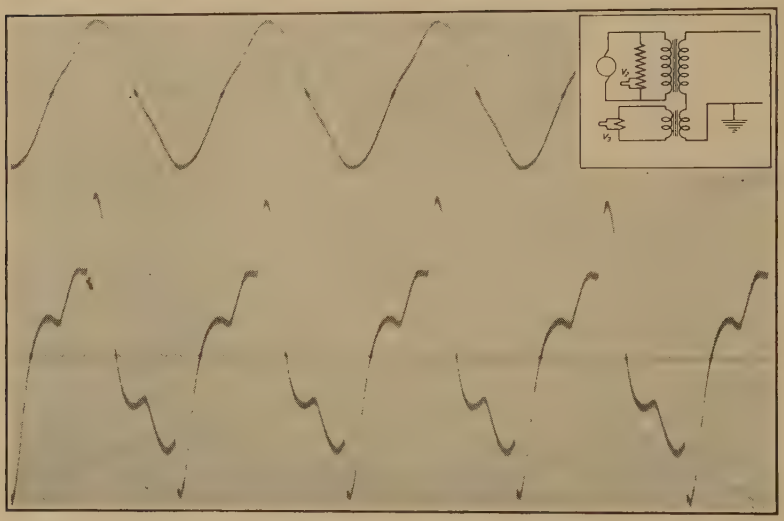


FIG. 160.—Corona. Variable condensance.
E = 22.5 kv.; length of line, 88.4 ft.; spacing, 4 in.; brass wire, No. 24 A.w.g.

of the successive voltage waves, when the instantaneous voltage gradient exceeds 29.8 kv. per cm., the required value for producing visual corona. As a consequence the corona produces harmonics in the condensance or charging current of the circuit. This is illustrated by the oscillograms in Figs. 159 and 160. If an alternating-current voltage of sine wave shape is impressed on a circuit having constant condensance the charging current would also follow the sine law. If the condensance, *the C circuit constant*, varies during the voltage cycle, a corresponding

change is produced in the wave shape of the charging current.

Problems and Experiments

1. Take oscillograms, similar to Figs. 147 to 151, showing the starting transients of transformers.

2. Take oscillograms showing the variable condensance of an arcing ground for direct and alternating currents on a transmission line.

3. Take oscillograms similar to Figs. 159 and 160, showing the change in condensance or charging current produced by corona.

4. Take an oscillogram similar to Fig. 151 showing the voltage across the terminals. Compare the operating voltage with the maximum value when the switch is opened.

5. Plot resistance-time curve for tungsten lamp from oscillograms in Fig. 144. Scale in ohms.

6. Plot resistance-time curve for carbon lamp from oscillograms in Fig. 145. Scale in ohms.

CHAPTER VIII

RESONANCE

Electric resonance phenomena have essentially permanent or stable characteristics but are closely related to, and frequently accompanied by, true electric transients. The conditions required for producing resonance and expressions for the frequency at which resonance occurs, in simple electric circuits, are referred to in Chap. V in connection with the derivation of the equations for the natural frequency of free oscillations. Resonance in an electric circuit implies a forced oscillation of energy between the magnetic and dielectric fields, during which the energy dissipated as heat by the Ri^2 and Ge^2 losses is supplied from some outside source. Distinction is usually made between *voltage resonance* occurring in series circuits, and *current resonance* that may be produced in parallel circuits.

Voltage Resonance.—In series circuits *voltage resonance occurs at that frequency of the impressed voltage for which the impedance of the circuit is a minimum.* In series circuits, as in Fig. 161, the impedance is a minimum when the condensive and inductive reactances are equal.

$$ {}_{\iota}x = {}_{c}x;\ 2\pi fL = \frac{1}{2\pi fC} \tag{275}$$

$$ f = \frac{1}{2\pi\sqrt{LC}} \tag{276}$$

$$ z = \sqrt{R^2 + ({}_{\iota}x - {}_{c}x)^2} = R \tag{277}$$

Frequently the assumption is made that a circuit is in resonance when the current and the impressed voltage are in phase, as illustrated by the vector diagram in Fig. 162. For straight series circuits the conditions required for unity

power factor of the power supplied to the circuit are identical with the requirements for minimum impedance, but in complex circuits or for current resonance in parallel circuits this is not always the case.

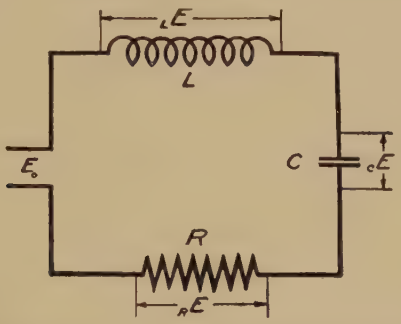


FIG. 161.—Series circuit for voltage resonance.

Equation (276) gives the optimum condition for resonance in series circuits for given values of the R, L and C, the circuit constants. Resonance phenomena are, however,

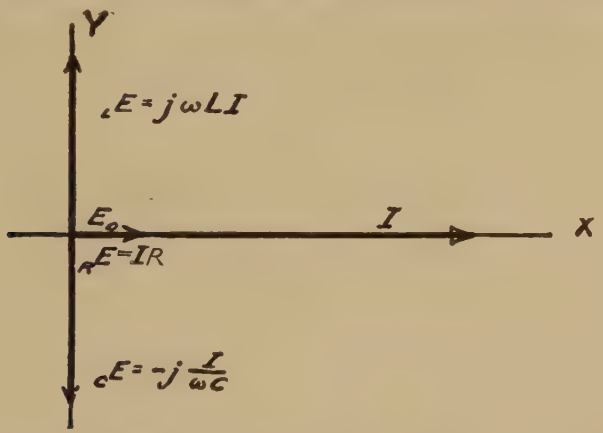


FIG. 162.—Vector diagram for voltage resonance in series circuit, Fig. 161.

not limited to the exact frequency determined by equation (276), but persist over a range of frequencies, more or less sharply defined, depending on the relative magnitude of the resistance and the inductive or condensive reactance. The voltage-frequency relation for given constant values

of R, L and C is shown in Fig. 163. The feature of special interest is the large increase in ${}_{L}E$ and ${}_{C}E$, the voltages across the inductance and the condensance under resonance conditions. If the resistance is small ${}_{L}E$ and ${}_{C}E$ may rise

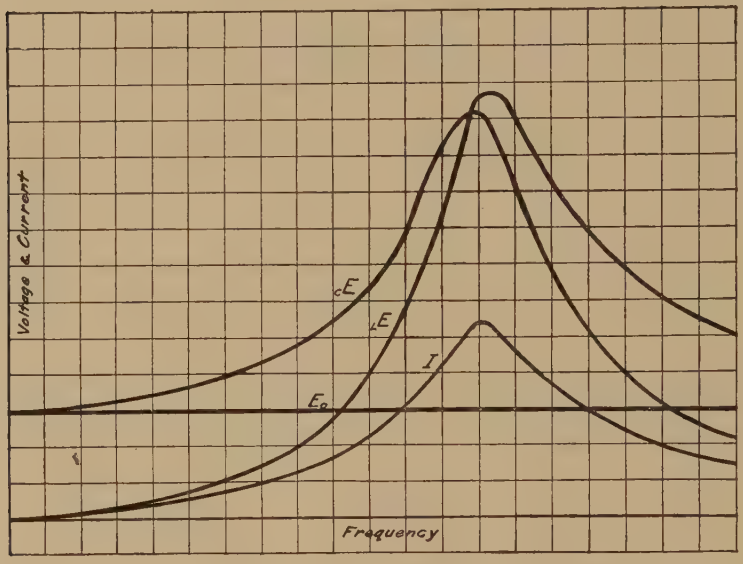


Fig. 163.—Voltage resonance for series circuit, as in Fig. 161.

to many times the value of the impressed voltage E_0. Voltage resonance in power circuits is undesirable, as the increase in voltage above the normal operating value endangers the insulation. The resistance in circuit is, in general, ample to hold the voltage within safe operating limits.

The effect on the *sharpness of resonance* of varying the resistance is illustrated by Fig. 164. The smaller the resistance the higher and sharper the voltage and current resonance peaks. The sharpness of resonance may be defined as the ratio of the inductive reactance or the condensive reactance at resonance frequency to the resistance in the circuit.

$$\text{Sharpness of resonance} = \frac{{}_{L}x}{R} = \frac{{}_{C}x}{R} \qquad (278)$$

Reactance Curves.—Curves in rectangular coördinates showing graphically the changes in magnitude of the inductive reactance and the condensive reactance produced by varying the frequency of the impressed voltage are of much value for giving a clear insight into resonance phenomena. The ordinates of the curves in Fig. 165 represent respectively the inductive reactance $_{L}x$, the condensive reactance $_{C}x$ and the total reactance x with the frequency of impressed voltage as the other variable represented

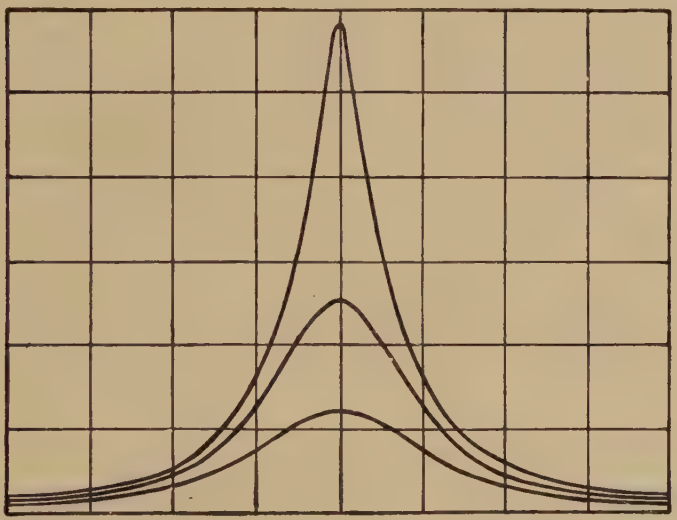


FIG. 164.—Resonance curves for series circuit with different resistances.

along the X-axis. Since resonance occurs when the impedance of the series circuit is a minimum, the resonance frequency is indicated by the intersection of the total reactance curve, in Fig. 165, with the X-axis.

Current Resonance.—Forced oscillatory transfer of energy between dielectric and magnetic fields is the basis of resonance phenomena in parallel circuits in much the same manner as in series circuits, but the resultant voltage and current values are different. In simple parallel circuits, as illustrated by Figs. 166 and 170, *current resonance occurs at that frequency of the impressed voltage for*

which the total admittance is a minimum. In discussions of resonance phenomena it is frequently assumed that the conditions for current resonance in parallel circuits are met when the inductive and condensive susceptances are equal, that is, when the impressed current and voltage are in phase. That this assumption is not in full accord with the above definition of current resonance for all

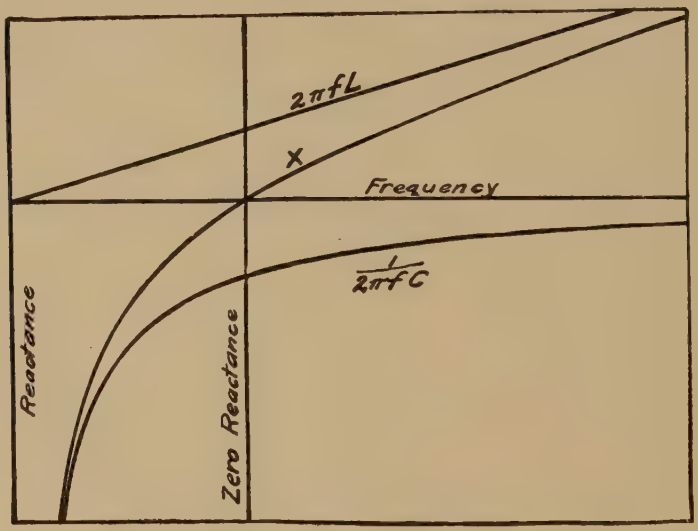


FIG. 165.—Reactance curves. Series circuit.

values of R in the circuits shown in Figs. 166 and 170 may readily be seen from the corresponding vector diagrams in Figs. 167 and 171. For the circuit in Fig. 166 current resonance occurs when ${}_{L}b = {}_{C}b$ under the condition that $R = O$. From the vector diagram in Fig. 167:

$$ {}_{L}\dot{I} = \dot{E}_0(g - j{}_{L}b) = \dot{E}_0\frac{R - j\omega L}{R^2 + \omega^2 L^2} \tag{279} $$

$$ {}_{C}\dot{I} = j{}_{C}b\dot{E}_0 = j\omega C\dot{E}_0 \tag{280} $$

$$ \dot{I} = {}_{L}\dot{I} + {}_{C}\dot{I} = \dot{E}_0[g + j({}_{C}b - {}_{L}b)] \tag{281} $$

$$ I = E_0\sqrt{g^2 + ({}_{C}b - {}_{L}b)^2} \tag{282} $$

$$ = E\sqrt{\left(\frac{R}{R^2 + \omega^2 L^2}\right)^2 + \left(\omega C - \frac{\omega L}{R^2 + \omega^2 L^2}\right)^2} \tag{283} $$

The total current I will be in phase with the impressed voltage E, if

$$_cb = {}_Lb;\ \omega C = \frac{\omega L}{R^2 + \omega^2 L^2} \tag{284}$$

Hence for unity power factor supply, the frequency for the circuit in Fig. 166,

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \tag{285}$$

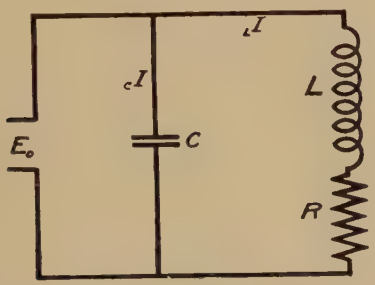


FIG. 166.—Parallel circuit for current resonance.

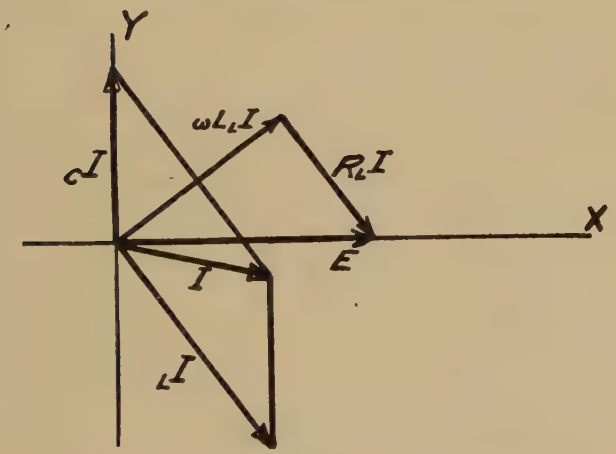


FIG. 167.—Vector diagram for circuit in Fig. 166.

For maximum current resonance the total admittance of the circuit must be a minimum and hence for constant impressed voltage E_0 the total current must be a minimum. Therefore, the resonance frequency may be obtained by

equating the first derivative of I to ω, L or C, as the case may be, in equation (283) to zero. Taking ω as the variable factor with R, L, C and E constants for the circuit in Fig. 166:

$$f = \frac{1}{2\pi}\sqrt{\left(\frac{1}{C^2L^2} + \frac{2R^2}{CL^3}\right)^{\frac{1}{2}} - \frac{R^2}{L^2}} \qquad (286)$$

Letting C be the variable factor with R, L, ω and E constant:

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \qquad (287)$$

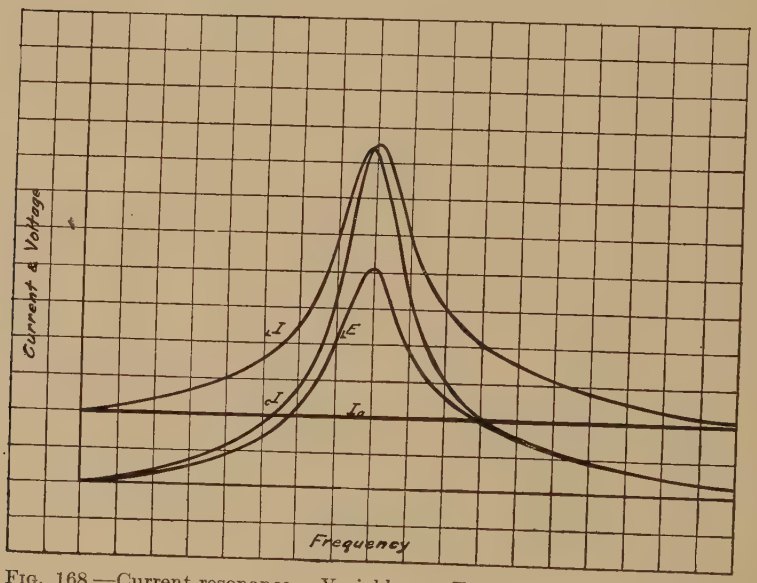


FIG. 168.—Current resonance. Variable ω. For Fig. 166, equation (286).

Letting L be variable with R, C, ω and E constants:

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{2LC} + \left(\frac{R^4}{L^4} + \frac{R^2}{CL^3} + \frac{1}{4L^2C^2}\right)^{\frac{1}{2}}} \qquad (288)$$

In a similar manner expressions may be obtained for *unity power factor frequency* and *maximum current resonance frequency* for ω, C or L respectively as the variable with the other factor constants for the circuit in Fig. 170.

$$_{L}\dot{I} = \dot{E}_0(g - j_{L}b) \tag{289}$$

$$_{C}\dot{I} = \dot{E}_0(G + j_{C}b) \tag{290}$$

$$\dot{I} = {}_{L}\dot{I} + {}_{C}\dot{I} = \dot{E}_0[(g + G) + j({}_{C}b - {}_{L}b)] \tag{291}$$

$$I = E_0\sqrt{\left(\frac{R}{R^2 + \omega^2L^2} + G\right)^2 + \left(\omega C - \frac{\omega L}{R^2 + \omega^2L^2}\right)^2} \tag{292}$$

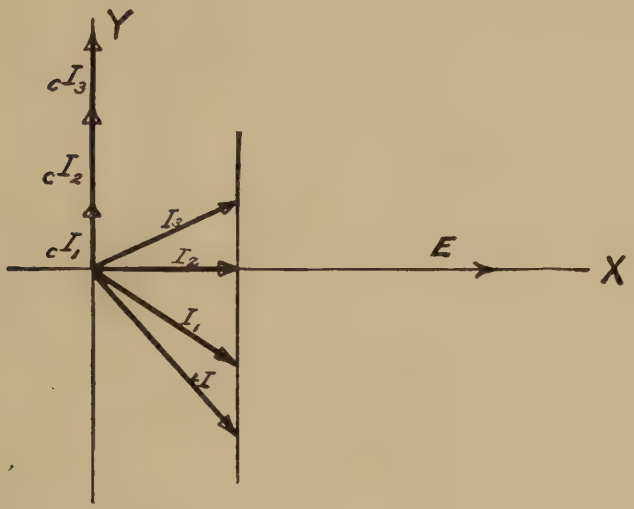


FIG. 169.—Vector diagram. Variable C. For Fig. 166, equation (287).

The total current I will be in phase with the impressed voltage E_0 if

$$_{C}b = {}_{L}b;\ \omega C = \frac{\omega L}{R^2 + \omega^2L^2} \tag{293}$$

Hence, the frequency required to give unity power factor for the circuit in Fig. 170 is the same as for Fig. 166.

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \tag{294}$$

The frequency for maximum current resonance if ω is variable while R, L, C, G and E_0 are constant, Figs. 170 and 171:

$$f = \frac{1}{2\pi}\sqrt{\left(\frac{1 - 2RG}{C^2L^2} + \frac{2R^2}{CL^3}\right)^{\frac{1}{2}} - \frac{R^2}{L^2}} \tag{295}$$

If C be the variable, while R, L, G, ω and E_0 are constant:

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \tag{296}$$

If L be the variable, while R, C, G, ω and E_0 are constant:

$$f = \frac{1}{2\pi}\sqrt{\frac{1 + 2RG}{2LC} + \left[\frac{R^4}{L^4} + \frac{R^2 + 2RG}{CL^3} + \frac{(1 + 2RG)^2}{4L^2C^2}\right]^{\frac{1}{2}}} \tag{297}$$

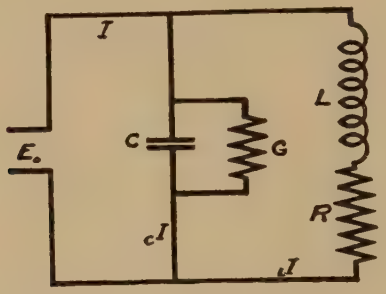


FIG. 170.—Parallel circuit with leaky condenser.

In tuning radio receiver sets resonance is obtained by varying C or L as expressed by equations (296) and (297).

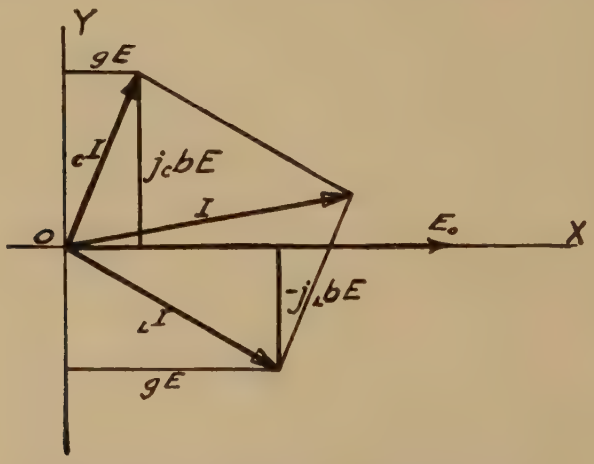


FIG. 171.—Vector diagram for circuit in Fig. 170.

Changes in the inductance by varying the number of turns also change the ohmic resistance but the conditions

required for equation (297) may be obtained experimentally for circuits in which the change in L may be produced by varying the mutual or self-induction between parts of the inductance in circuit.

The smaller the resistance in the resonating circuit the greater the increase in the resonance current and voltage. *Resonance phenomena are of commercial importance only when the resistance in circuit is small as compared to the inductance and condensance.*

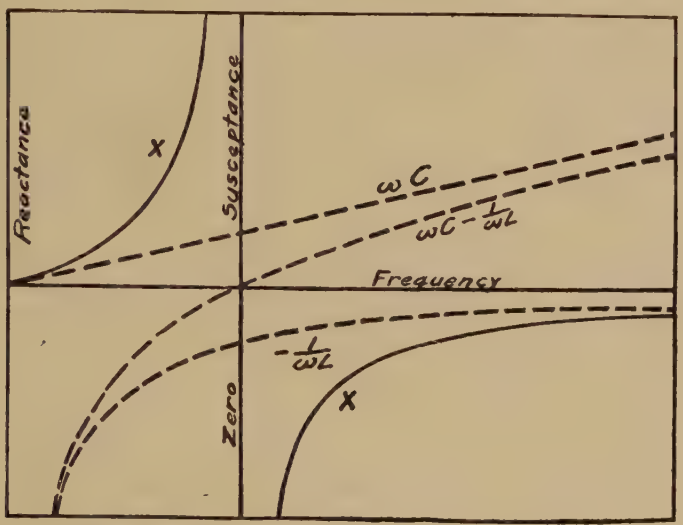


FIG. 172.—Susceptance curves for parallel circuit.

In most cases, and particularly those of greatest importance, the resistance is negligibly small. If R and G are taken equal to zero all the resonance frequency equations (295) to (297) become identical in form.

Resonance frequency, massed circuit constants (approximate value):

$$f = \frac{1}{2\pi\sqrt{LC}} \qquad (298)$$

In commercial work in power systems equation (298) is in general use, giving with sufficient accuracy the resonance

for simple circuits having *massed condensance, inductance* and *resistance.*

For distributed circuit constants, as in long transmission lines, the space distribution of the voltage and current waves must be taken into consideration; the approximate resonance frequency is given by equation (299), as explained in Chap. VI on Transmission-line Oscillations.

Resonance frequency, uniformly distributed circuit constants (approximate value):

$$f = \frac{1}{4\sqrt{LC}} \tag{299}$$

In power circuits resonance conditions must be avoided or the resistance in circuit be sufficiently large to prevent any marked increase due to resonance in the current and voltage.

Coupled Circuits.—Resonance phenomena are of fundamental importance in the operation of radio communication apparatus. The circuits in commercial use are more complex than the forms discussed above but may be considered as combinations of simple circuits. In general, the component simple circuits have certain parts in common.

The couplings or connections may be made in a number of ways. For two-circuit apparatus the coupling is generally made in one of the following ways:

1. By direct connection across an inductance coil. Direct coupling as in Fig. 173.

2. By magnetic induction. Inductive or magnetic coupling as in Fig. 174.

3. By dielectric induction. Condensive, capacitative or dielectric coupling as in Fig. 175.

The inductive interaction of the voltages and currents in two resonating coupled circuits and the transfer of the oscillating energy between the primary and secondary circuits are illustrated by the oscillograms in Figs. 176 to 181. The oscillations of the energy between the dielectric and magnetic fields of each circuit are combined with a

rapid to and fro transfer of the energy between the magnetically or dielectrically coupled circuits. In Fig. 176

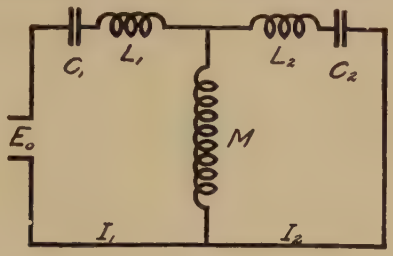


Fig. 173.—Direct coupling.

the energy was initially stored in the condenser in the primary circuit. By closing the switch oscillations are set up

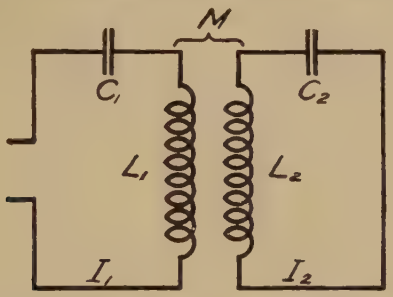


Fig. 174.—Inductive or magnetic coupling.

between the dielectric and magnetic fields in both the primary and secondary circuits, and these are combined

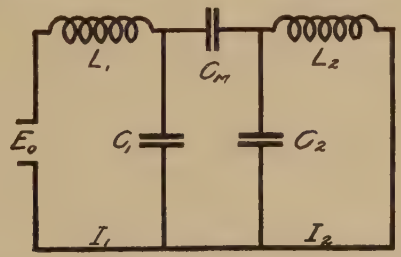


Fig. 175.—Condensive or dielectric coupling.

with a rapid to and fro transfer of the energy between the two circuits. It should be noted that the timing wave v_1 represents 200 cycles per sec. in Figs. 176 and 177

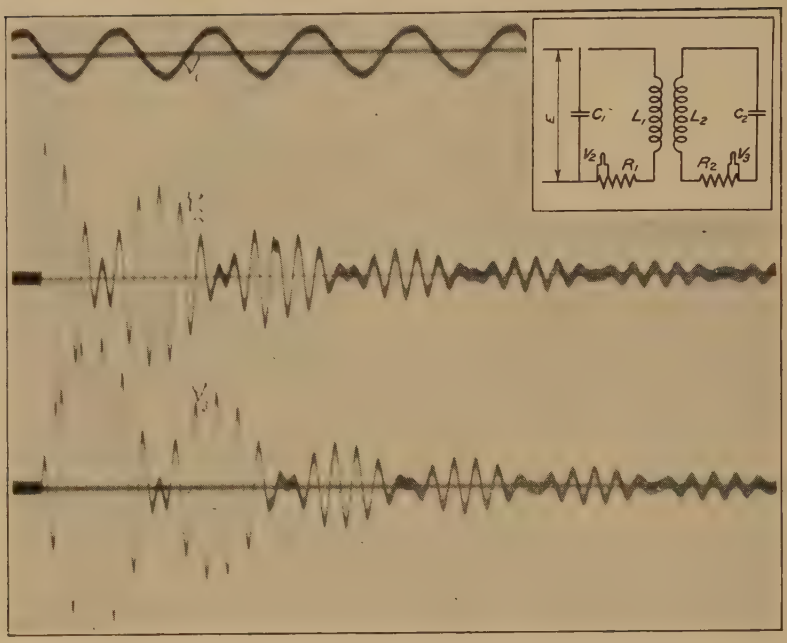


FIG. 176.—Transient oscillations of inductively coupled circuits.
E = 500 volts; $L_1 = L_2$ = 0.237 henry; $C_1 = C_2$ = 0.15 mf.; R_1 = 12.6 ohms; R_2 = 13.0 ohms; K, coefficient of coupling = 14 per cent; $v_1 = 200\sim$.

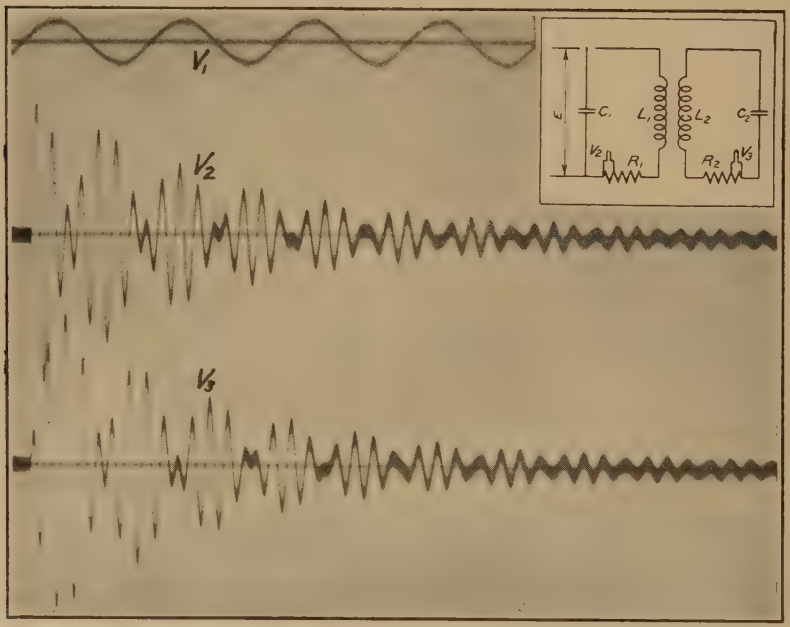


FIG. 177.—Transient oscillations in inductively coupled circuits.
E = 500 volts; L_1 = 0.133 henry; L_2 = 0.139 henry; $C_1 = C_2$ = 0.12 mf.; R_1 = 9.3 ohms; R_2 = 9.8 ohms; K, coefficient of coupling = 15 per cent; v_1 = 200~.

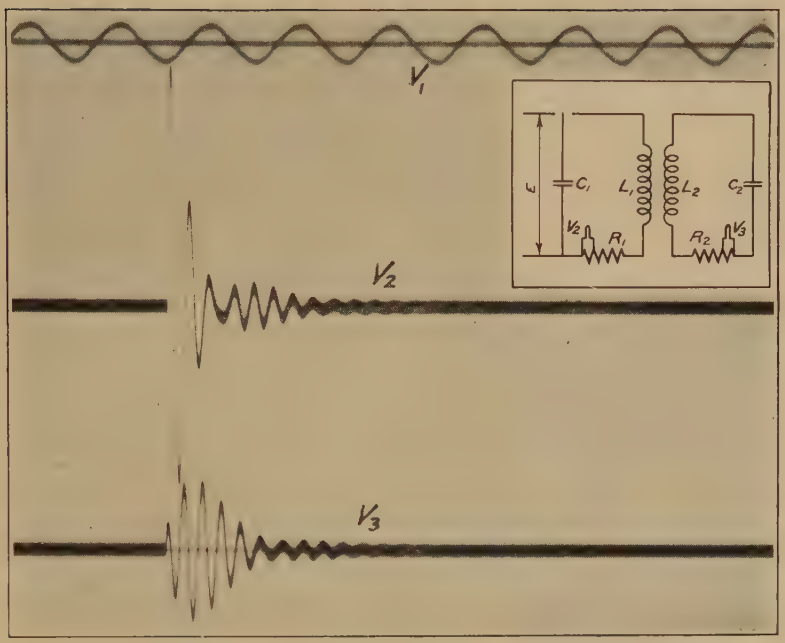


FIG. 178.—Transient oscillations in inductively coupled circuits.
E = 475 volts; $L_1 = L_2$ = 0.237 henry; $C_1 = C_2$ = 0.45 mf.; R_1 = 85 ohms; R_2 = 80 ohms; K, coefficient of coupling = 14 per cent.; $v_1 = 100\sim$.

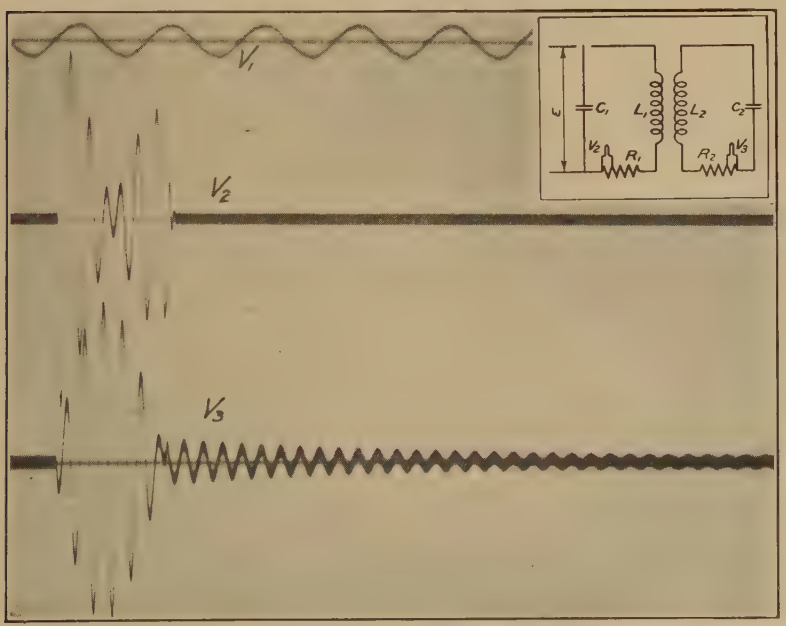


FIG. 179.—Transient oscillations in inductively coupled circuits. Primary circuit opened at peak of second beat.
E = 375 volts; $L_1 = L_2$ = 0.237 henry; $C_1 = C_2$ = 0.45 mf.; R_1 = 12.6 ohms; R_2 = 13.0 ohms; K, coupling coefficient = 14 per cent; $v_1 = 100\sim$.

instead of 100 cycles as in Figs. 35 to 37 and in Figs. 178 to 181. The oscillogram in Fig. 176 shows that the frequency of oscillation between the magnetic and dielectric fields in the primary was approximately 1,000 cycles per sec., while the beats or frequency of transfer between the circuits was very nearly 160 cycles per sec. That

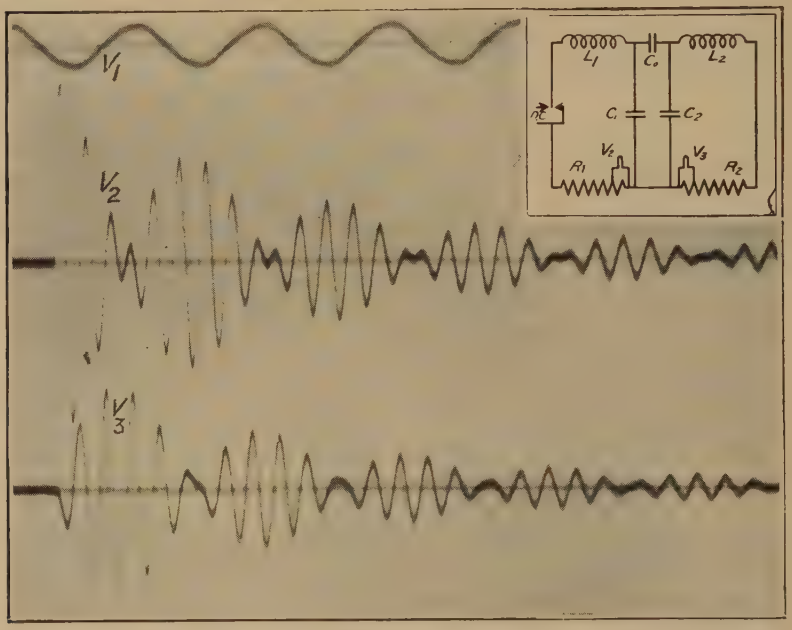


FIG. 180.—Transient oscillations in condensively or dielectrically coupled circuits.

E = 118 volts; L_1 = 0.152 henry; L_2 = 0.102 henry; C_1 = 0.69 mf.; C_2 = 1.10 mf.; C_0 = 0.32 mf.; R_1 = 9.82 ohms; R_2 = 4.43 ohms; $v_1 = 100\sim$.

is, the time required for the transfer of the energy from the primary to the secondary through the magnetic coupling and back again was slightly more than six complete oscillations between the magnetic and dielectric fields of the primary circuit. Due to the Ri^2 losses the beats rapidly decrease in magnitude and practically all of the energy was dissipated into heat in $\frac{1}{30}$.

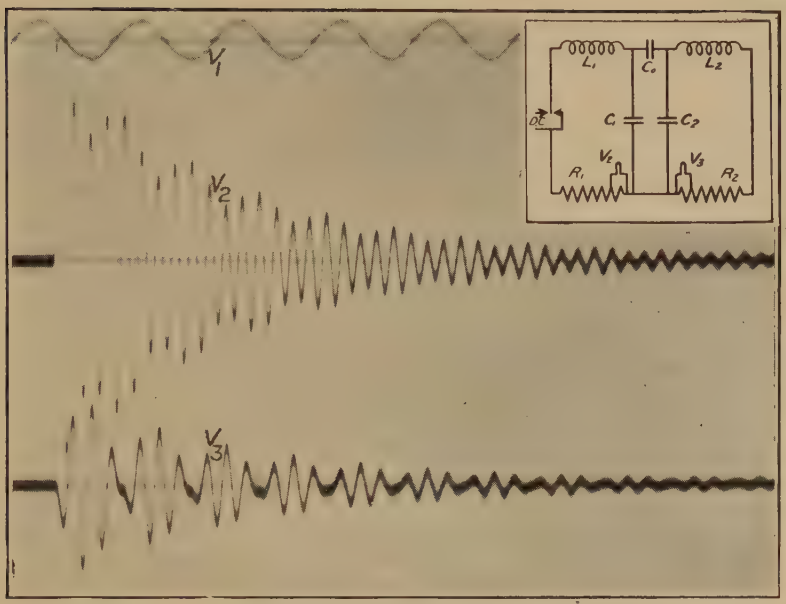


FIG. 181.—Transient oscillations in dielectrically coupled circuits.
E = 118 volts; L_1 = 0.10 henry; L_2 = 0.102 henry; C_1 = 0.68 mf.; C_2 = 1.1 mf.; C_0 = 0.32 mf.; R_1 = 3.8 ohms; R_2 = 4.43 ohms; v_1 = 100∼.

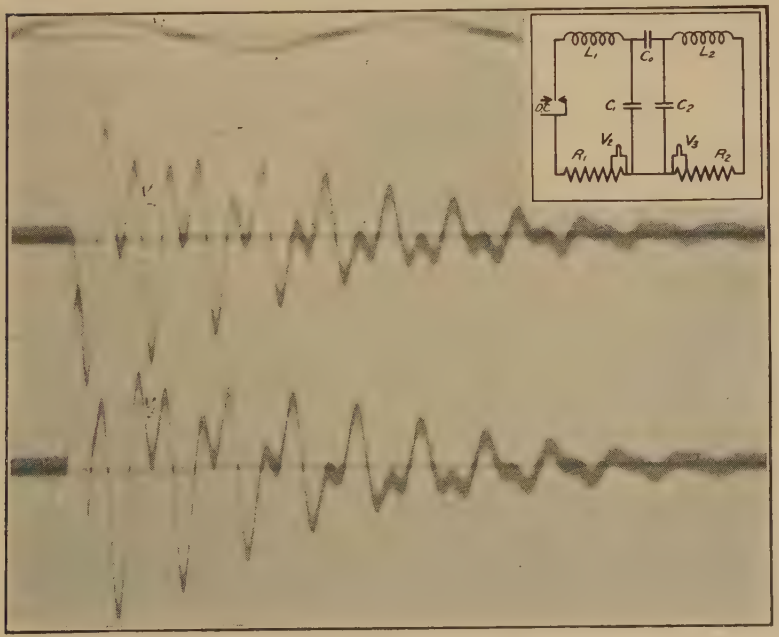


FIG. 182.—Dielectrically or condensively coupled circuits.
E = 120 volts direct current; L_1 = 0.10; L_2 = 0.11;; C_1 = C_2 = C_3 = 0.5 mf.; R_1 = R_2 = 14.5 ohms; v_1 = 60∼.

Similar oscillograms showing the oscillatory transfer of energy between the primary and secondary of dielectrically coupled circuits are shown in Figs. 180, 181 and 182.

Coupling Coefficient.—In coupled circuits as in Figs. 176 and 179, the interaction will depend on what part of the total magnetic flux interlinks both circuits. The degree

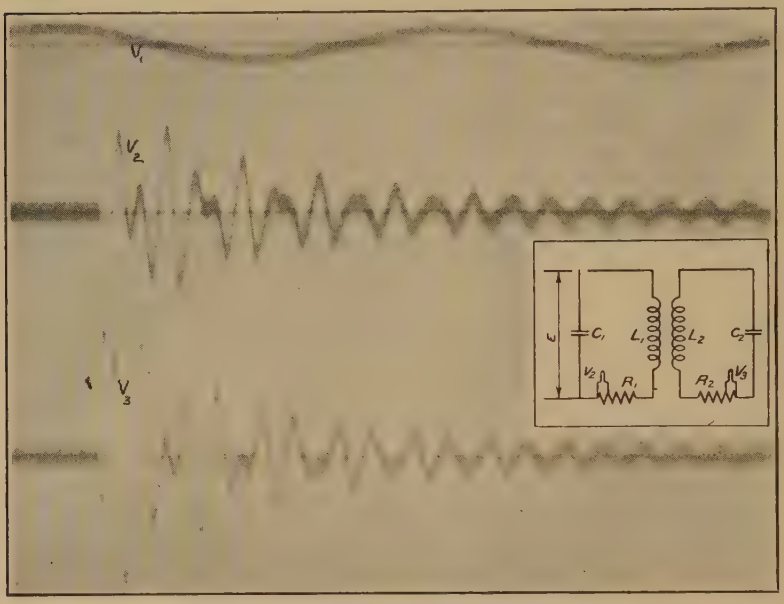


FIG. 183.—Inductively coupled circuits.

E = 120 volts direct current; L_1 = 0.0470 henry; L_2 = 0.0505 henry; C_1 = 1.0 mf.; C_2 = 1.0 mf.; R_1 = 10.7 ohms; R_2 = 10.4 ohms. Coefficient of coupling, K = 38.4 per cent; $v_1 = 60\sim$.

of coupling which is often termed "loose" or "close," depending on whether a small or a large fraction of the flux interlinks both circuits, is quantitatively expressed by the coupling coefficient. This is defined as the ratio of the mutual reactance to the square root of the product of the primary and secondary circuit reactances.

Inductive coupling coefficient, Fig. 174:

$$\kappa = \frac{{}_L x_m}{\sqrt{{}_L x_1 \, {}_L x_2}} = \frac{M}{\sqrt{L_1 L_2}} \qquad (300)$$

M = mutual inductance.

L_1 = inductance of primary with the secondary open or removed.

L_2 = inductance of secondary with the primary open or removed.

Condensive coupling coefficient, Fig. 175.

$$\kappa = \frac{{}_c x_m}{\sqrt{{}_c x_1 \, {}_c x_2}} = \frac{\sqrt{C_1 C_2}}{C_m} = \sqrt{\frac{C_a C_b}{(C_a + C_m)(C_b + C_m)}} \qquad (301)$$

C_m = condensance in common condenser.

C_a = condensance in primary circuit.

C_b = condensance in secondary circuit.

$C_1 = \dfrac{C_a C_m}{C_a + C_m}$ = total condensance in primary.

$C_2 = \dfrac{C_a C_m}{C_b + C_m}$ = total condensance in secondary.

Multiplex Resonance.—In complex circuits or series of double-energy loops the conditions for resonance may be satisfied for more than one frequency of the impressed voltage. The degrees of freedom, or the number of frequencies at which resonance may occur, depend on the number and interconnection of the elemental double-energy circuits in the system. Thus, a transmission line having uniformly distributed R, L, G and C, and hence to be considered as consisting of an infinite series of infinitesimal double-energy circuits, would resonate for the fundamental frequency of the line as a unit and for any multiple or harmonic of the fundamental frequency. As the line constants are not perfectly constant and the distribution of R, L, G and C not quite uniform, resonance is limited to the fundamental and a few of the lower harmonics.

Resonance Growth of Decay. Decrement.—As stated in the beginning of this chapter, resonance in electric circuits implies a forced oscillation of energy between magnetic and dielectric fields, at such frequencies of the impressed voltage as to make the total impedance or

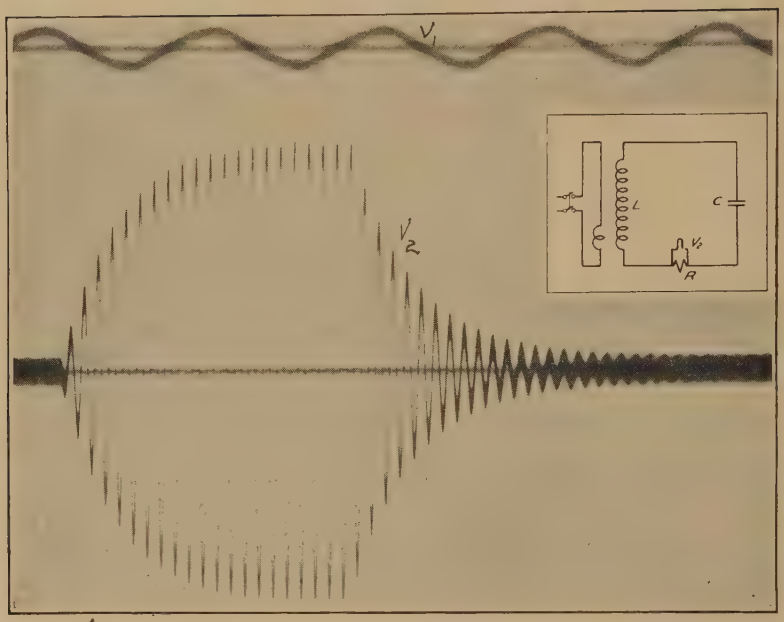


FIG. 184.—Cumulative resonance. Initial and decay transients
R = 2.18 ohms; L = 0.0175 henry; C = 1.018 mf.; f = 1,200~; v_1 = 100~

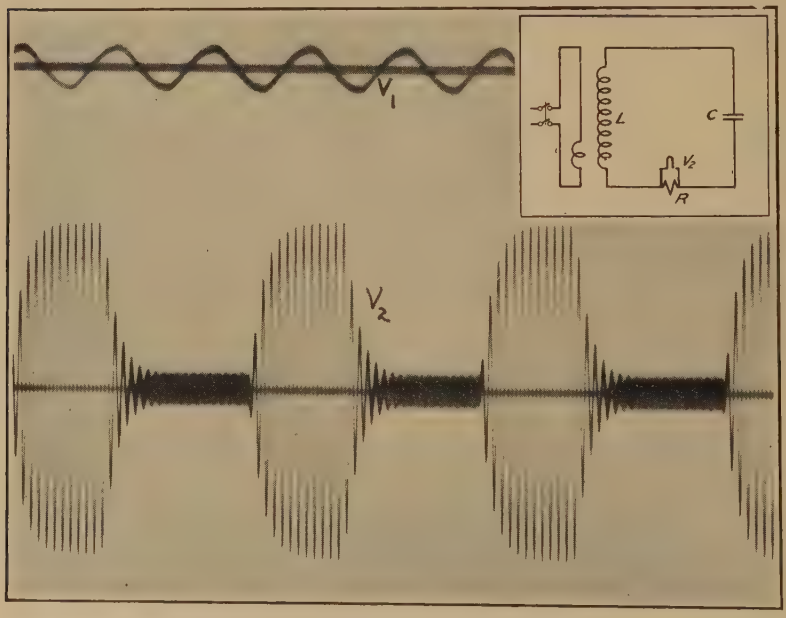


FIG. 185.—Resonance in high-speed signaling.
L = 0.0175 henry; C = 1.018 mf.; R = 17.1 ohms; v_1 = 100~.

admittance a minimum. To supply the resonating circuit with the oscillatory energy necessitates a transient starting period during which the amplitude of each oscillation is greater than the one preceding. For systems having constant finite circuit constants in which the resonance phenomena reach permanent values, the growth of the transient follows the exponential law. This increase in

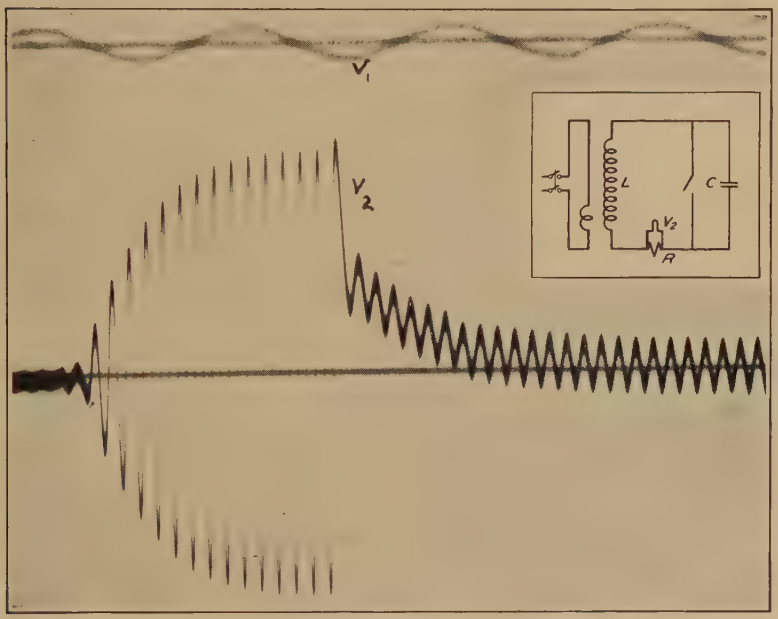


FIG. 186.—Cumulative resonance limited by spark-gap short circuit. $L = 0.0175$ henry; $C = 1.085$ mf.; $R = 3.2$ ohms; $v_1 = 100\sim$.

the magnitude of the oscillations during the starting period is illustrated by the oscillograms in Figs. 184 to 186. In these oscillograms the power supply was cut off when the resonance had reached the permanent stage. The decay parts of the oscillograms in Figs. 184 and 185, represent, therefore, free oscillations with a decrease in amplitude as the electric energy is dissipated into heat.

In Fig. 186 the starting period is of the same form as in Fig. 184 or 185, but not the decay stage. It is evident from

the circuit connections that the decay of the resonating currents or voltages will differ in shape depending at what instant in the cycle the short circuit occurs.

The decrement of a resonating circuit corresponds to the dissipation constant of an oscillating circuit. The numerical value of the decrement δ may be found from the circuit constants and the resonant frequency by equation (302).

$$\delta = \frac{R}{2fL} \tag{302}$$

Thus for Fig. 184 the decrement $\delta = 0.052$.

Problems and Experiments

1. Take oscillograms showing the transients accompanying the growth and decay of cumulative resonance in circuits similar to Figs. 184, 185 and 186.

2. Take oscillograms of the transient oscillations of two inductively coupled circuits similar to Figs. 176 to 179.

3. Take oscillograms of the transient oscillations in two dielectrically coupled circuits similar to Figs. 180 to 183.

CHAPTER IX

OSCILLOGRAMS AND KLYDONOGRAMS

In the preceding chapters the fundamental principles of electric transient phenomena are illustrated by a number of oscillograms, many of which the student should reproduce in order to gain the necessary appreciation of the quantitative value of the factors involved. The laboratory work in the course, however, should not be restricted to the reproduction of oscillograms appearing in the text for which quantitative data are provided, or to the taking of other oscillograms that merely illustrate the fundamental principles, for while the gaining of clear concepts of the basic laws of transient electric phenomena is of primary importance, training in applying the principles to practical engineering problems is likewise an essential part of the work. Ample material for this purpose is available in all electrical engineering laboratories. The oscillograms in this chapter, Figs. 187 to 209, which were selected from the laboratory reports of students in the introductory course in electric transients, may be taken as typical examples. The students were required to outline the problem, to select the necessary apparatus and instruments, to make preliminary calculations and to predict the form and shape of the transients to be recorded. They made the adjustments on the oscillograph, obtained experimentally the recorded quantitative data, took the oscillograms, developed the films and prepared a report on the transients photographically recorded by the oscillograph. Each oscillogram represents a separate problem to be analyzed on the basis of the principles discussed in the preceding chapters.

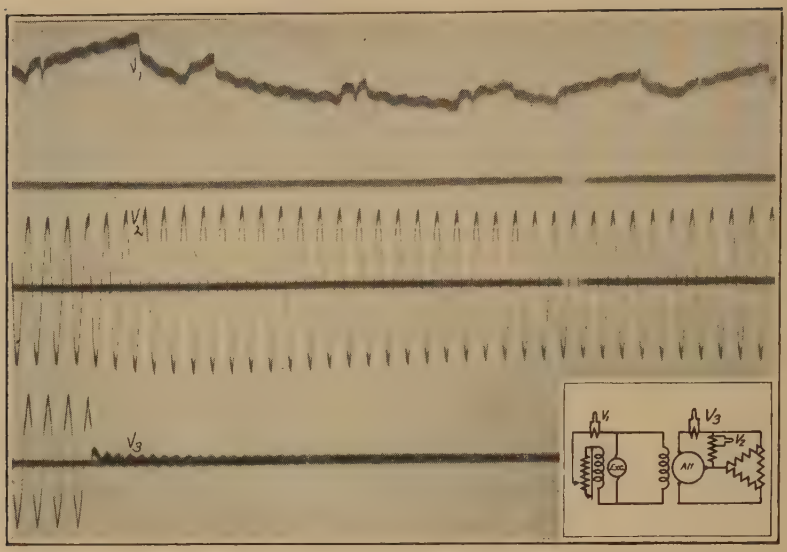


FIG. 187.—T. A. regulator. Full load to no load.

Line voltage = 120 volts. Line current = 25–0 amp. Field current, 3 amp. Exciter field, 1.1 amp. $f = 60\sim$.

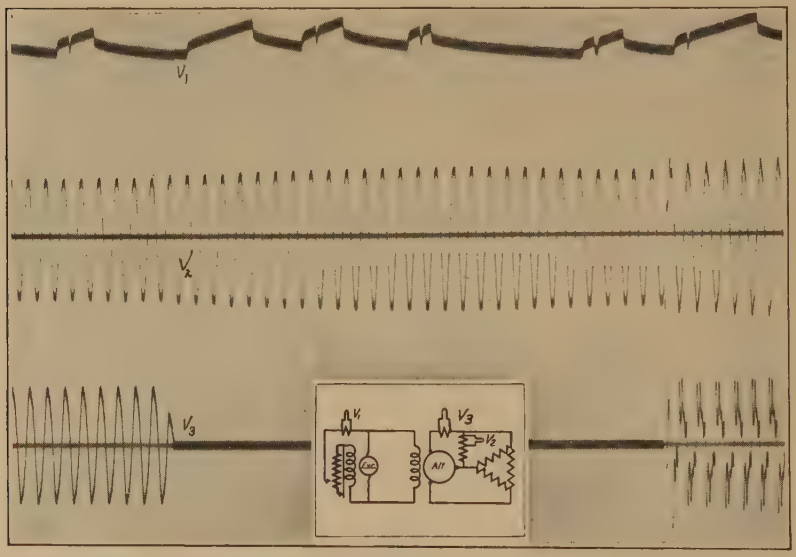


FIG. 188.—T. A. regulator. Inductive load—no load—condensive load.

Line voltage = 120 volts. Line current, 5–0–5 amp. Field current, 3 amp. Exciter field current, 1.0 amp. $f = 60\sim$.

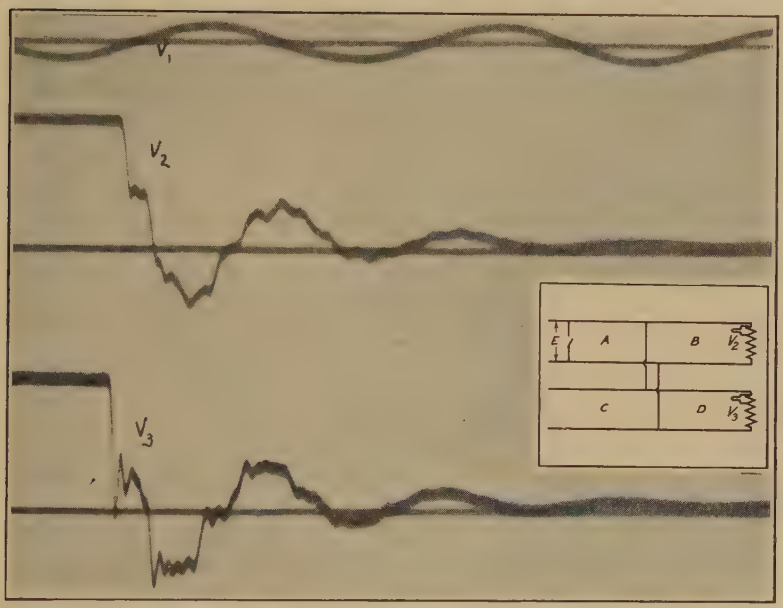


FIG. 189.—Electric oscillations in a transmission-line network. The four line sections *A*, *B*, *C* and *D* have the following circuit contents:

A	*B*	*C*	*D*	
21.0	31.4	34.6	14.8	= *R* in ohms.
0.172	0.258	0.272	0.116	= *L* in henrys.
1.22	1.84	2.14	0.92	= *C* in microfarads.
80	120	140	60	= equivalent length in miles.

$v_1 = 100\sim$.

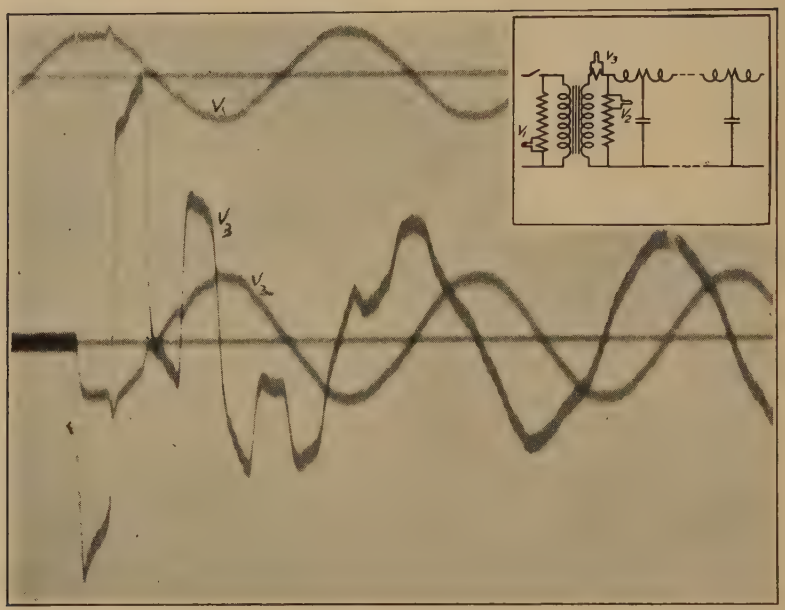


FIG. 190.—Switching A.C. voltage on transmission line with transformers. Receiver end open.

E = 100 volts; I = 0.53 amps.; f = 60. Line constants: L = 0.41 henry; C = 2.725 mfs.; R = 50.69 ohms. Transformers 3 kva.; ratio 5.1.

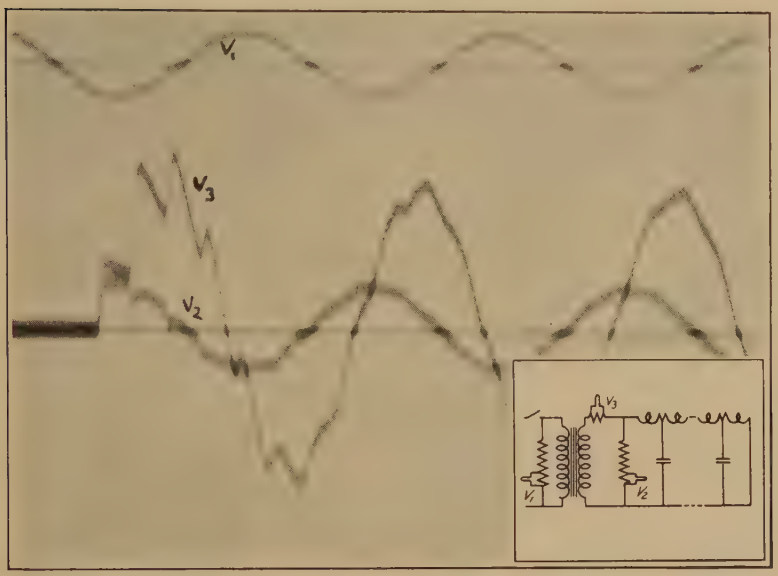


FIG. 191.—Switching A.C. voltage on transmission line and step up transformer. Receiver end short circuited.

I = 1.9 amps.; f = 60. Line constants: L = 0.41 henry; C = 2.725 mfs.; R = 50.69 ohms. Transformer: 3 kva.; ratio 5:1.

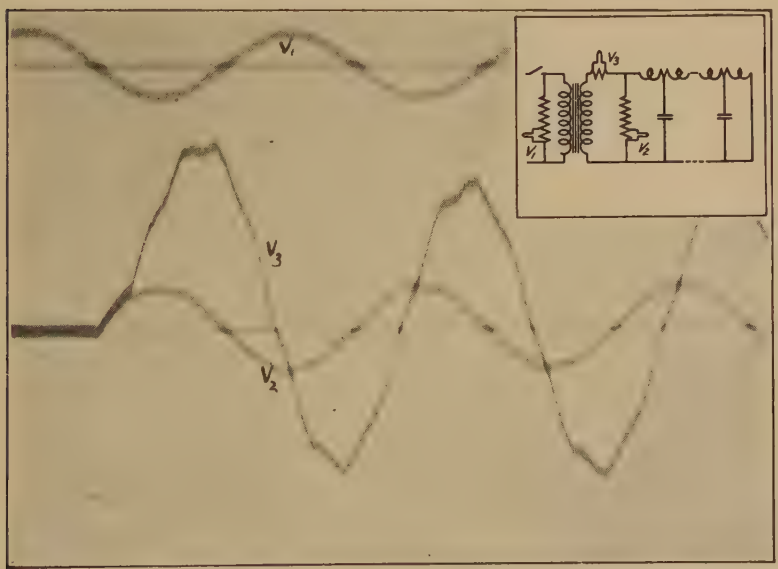


FIG. 192.—Switching A.C. voltage on transmission line and step-up transformer. Same data as in Fig. 191 except the switch was closed at another point on the voltage wave.

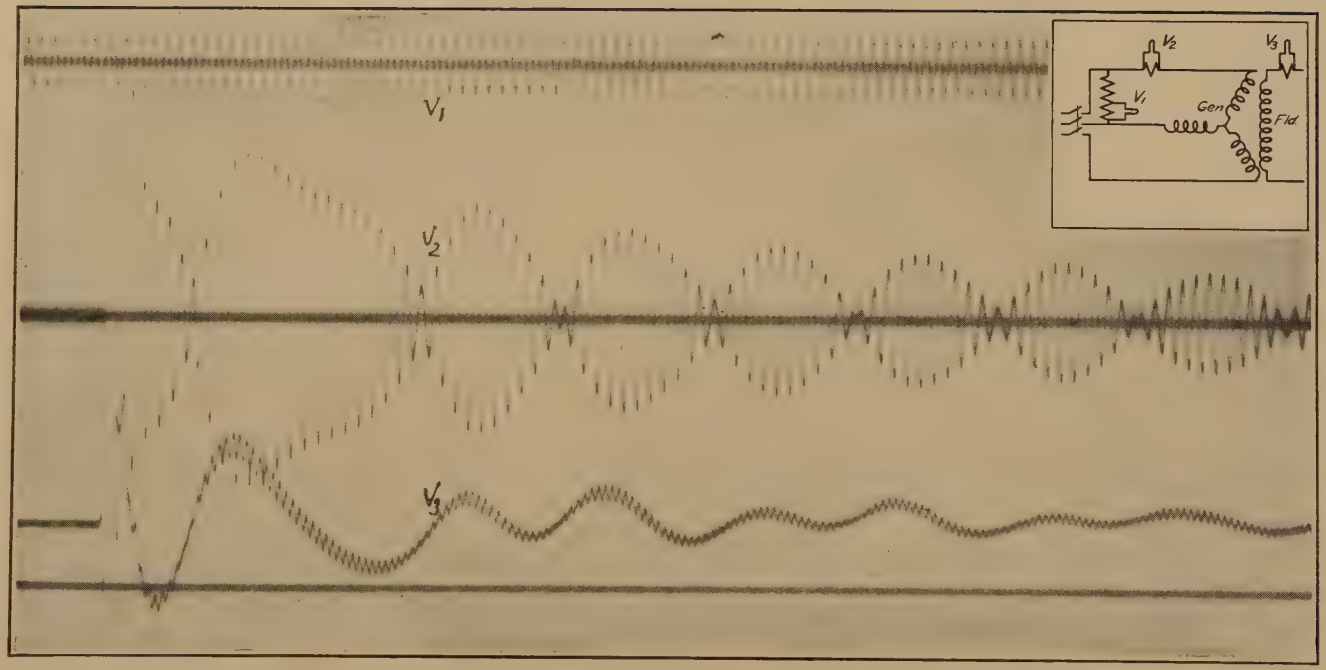


FIG. 193.—Two 7.5 kva. alternators thrown together while not running in synchronism.
E_{line} = 231 volts; E_{motor} = 206 volts; $v_1 = 60\sim$; I_f = 2.6 motor amperes; I_g = 2.5 amps.

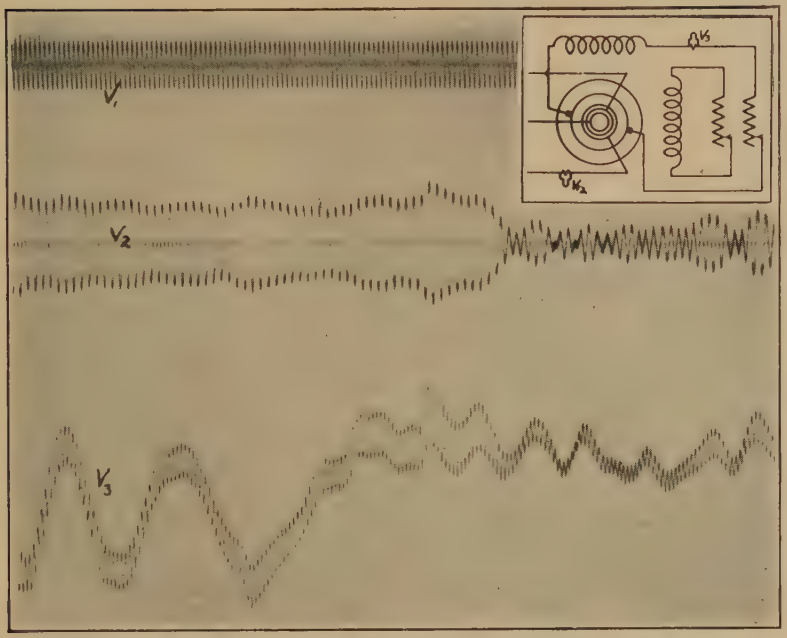


FIG. 194.—Fynn-Weichsel motor pulling into step.

E = 235 volts; I_{line} = 6.75 amps.; $I_{o.c.}$ = 7.5 amps., current in stator winding excited from commutated rotor winding; $v_1 = 100\sim$.

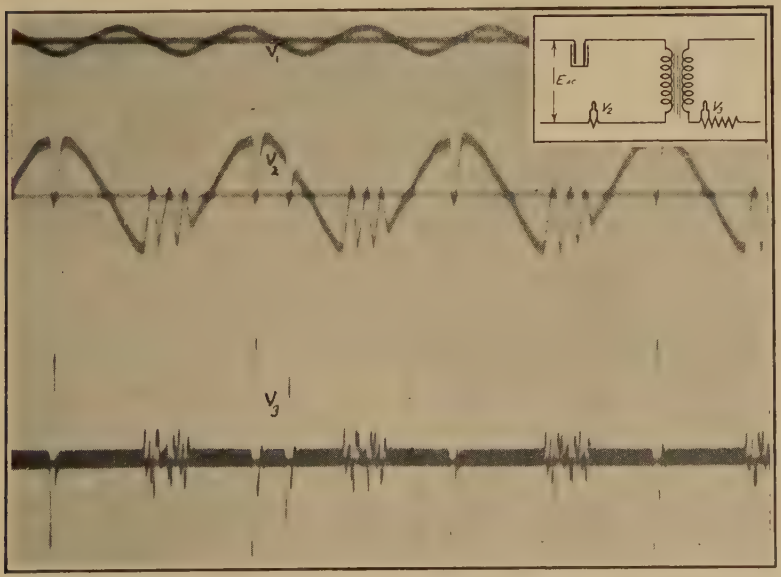


FIG. 195.—Electrolytic interrupter on an induction spark coil for A.C. circuit.

E = 120 volts; I = 8 amps; $v_1 = 100\sim$.

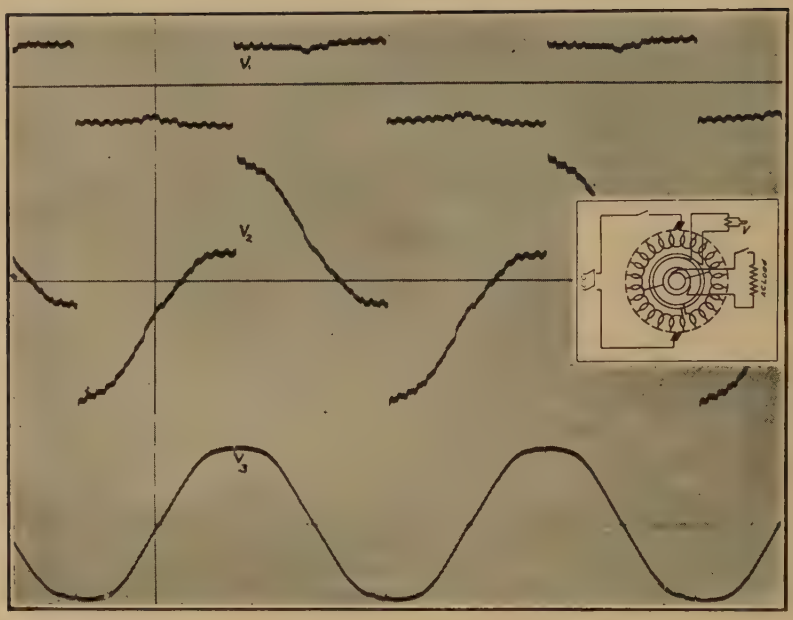


FIG. 196.—Current components in rotary converter. Vibrator inserted in armature winding next to slip ring tap.

v_1 = D.C. only, v_2 = A.C. and D.C., v_3 = A.C. only. Power factor 100 per cent. Machine operated as an inverted converter.

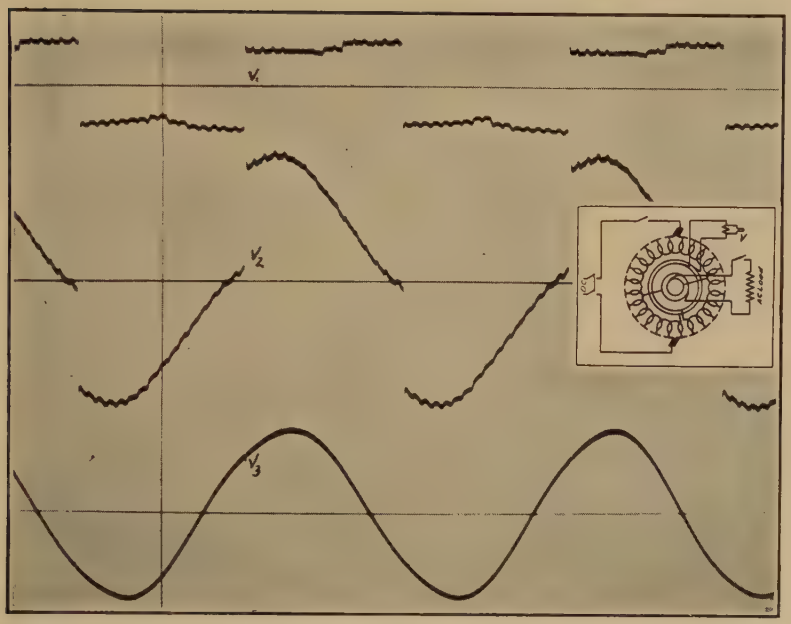


FIG. 197.—Current components in rotary converter. Vibrator inserted in armature winding next to slip ring tap.

v_1 = D.C. only, v_2 = A.C. and D.C., v_3 = A.C. only. Power factor 75 per cent current lagging. Machine operated as an inverted converter.

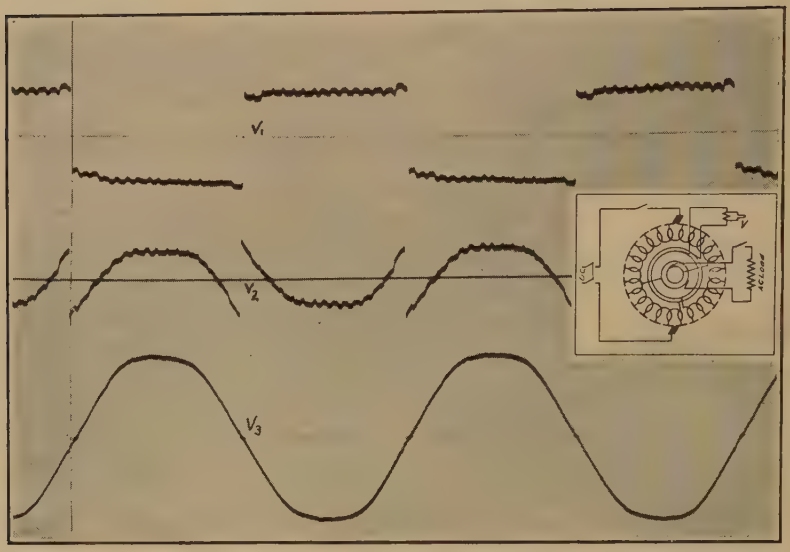


FIG. 198.—Current components in rotary converter. Vibrator inserted in armature winding at point midway between slip ring taps.

v_1 = D.C. only, v_2 = A.C. and D.C., v_3 = A.C. only. Power factor 100 per cent. Machine operated as an inverted converter.

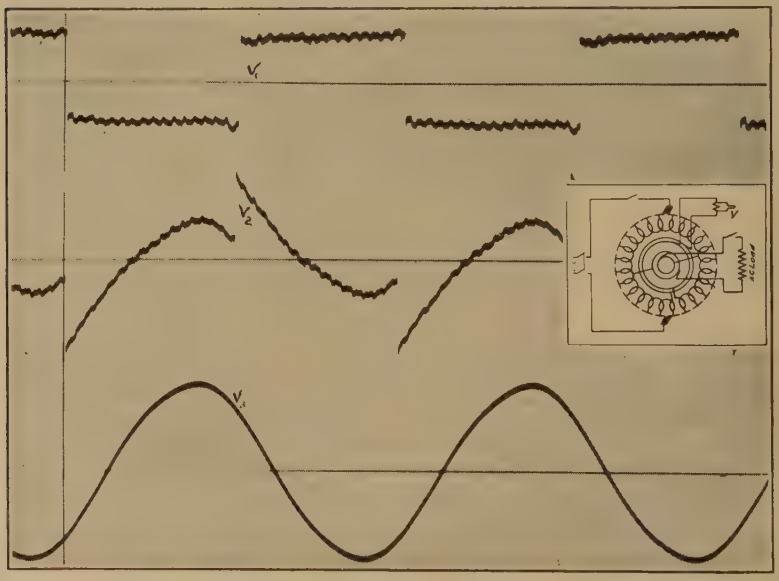


FIG. 199.—Current components in rotary converter. Vibrator inserted in armature winding at point midway between slip ring taps.

v_1 = D.C. only, v_2 = A.C. and D.C., v_3 = A.C. only. Power factor 75 per cent, current lagging. Machine operated as an inverted converter.

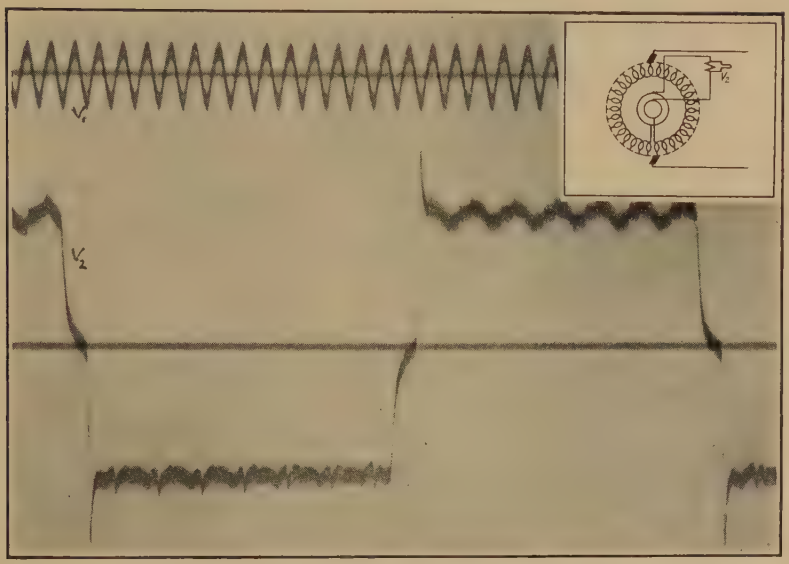


FIG. 200.—Commutation, D.C. generator. No load.
I_a = 0.5 amps.; I_f = 1.75 amps.; speed, 160 r.p.m.; v_1 = 200 ∼.

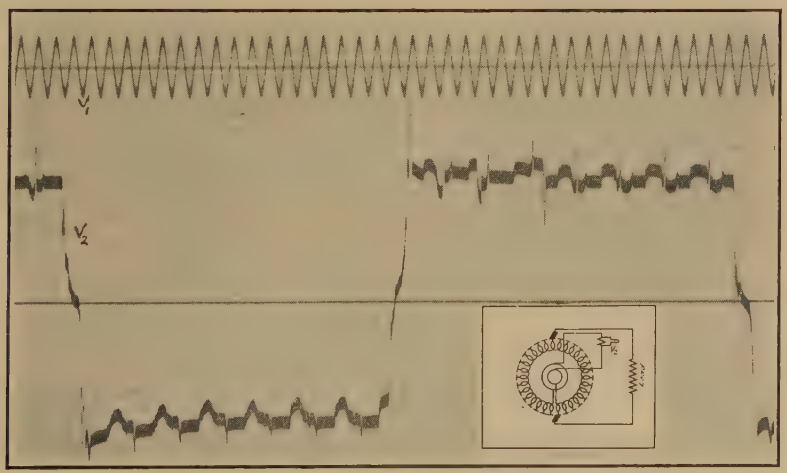


FIG. 201.—Commutation, D.C. generator. Load.
I_a = 17.6 amps.; I_f = 1.75 amps.; v_1 = 200 ∼.

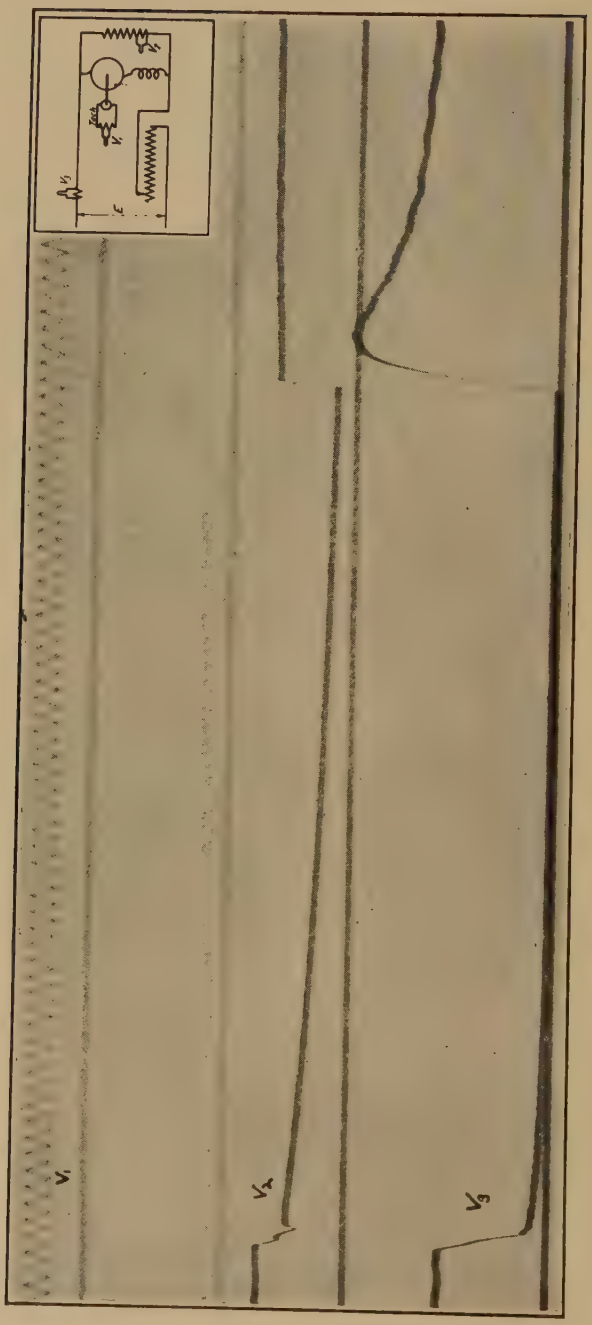


FIG. 202.—Railway motor at road crossing 50 per cent load. Without controller
E = 125 volts; I = 20 amps., v_1 = 100~.

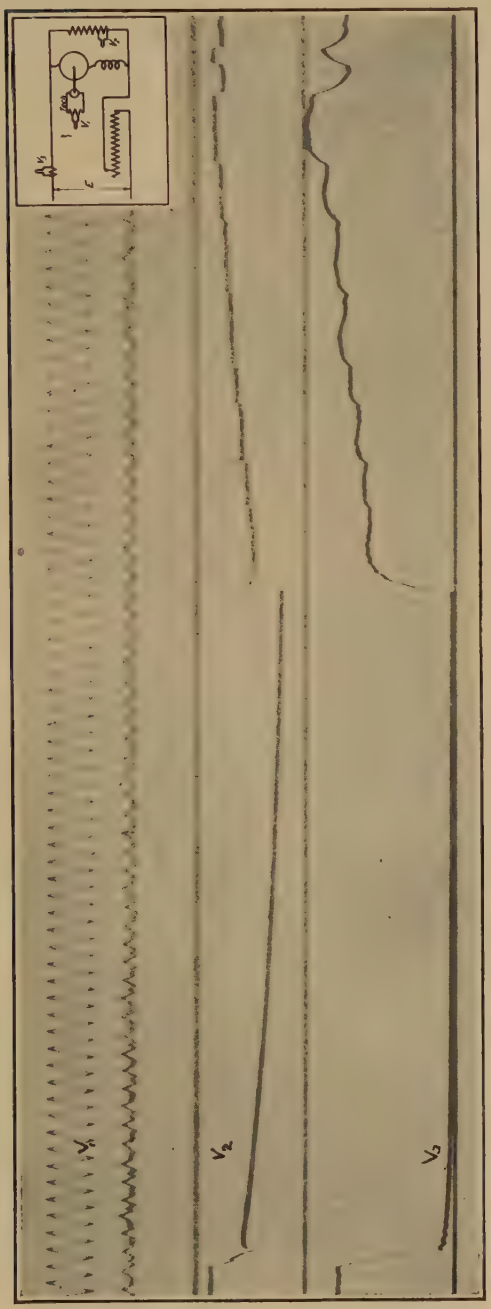


FIG. 203.—Railway motor at road crossing 100 per cent load. With controller.
E = 125 volts; I = 40 amps.; v_1 = 100 ∼.

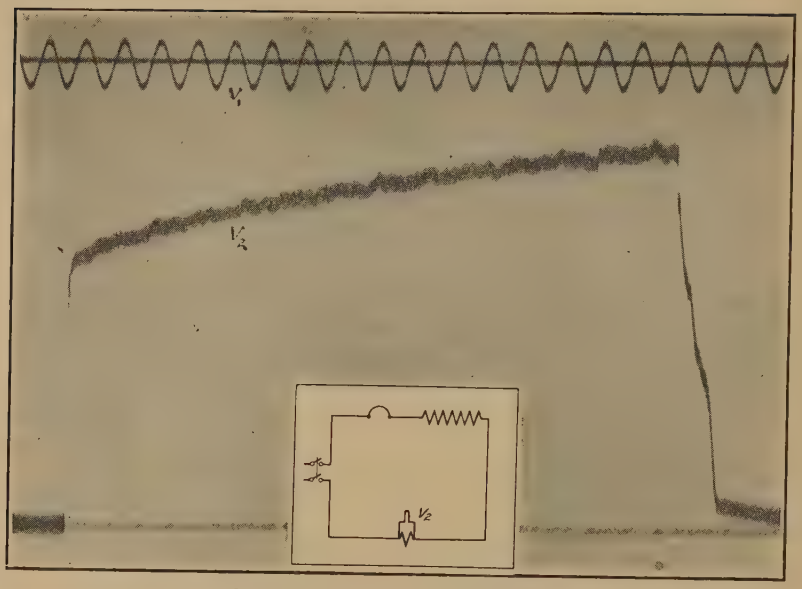


FIG. 204.—D.C. circuit breaker.
E = 128 volts; I = 21.3 amps.; v_1 = 60 ∼.

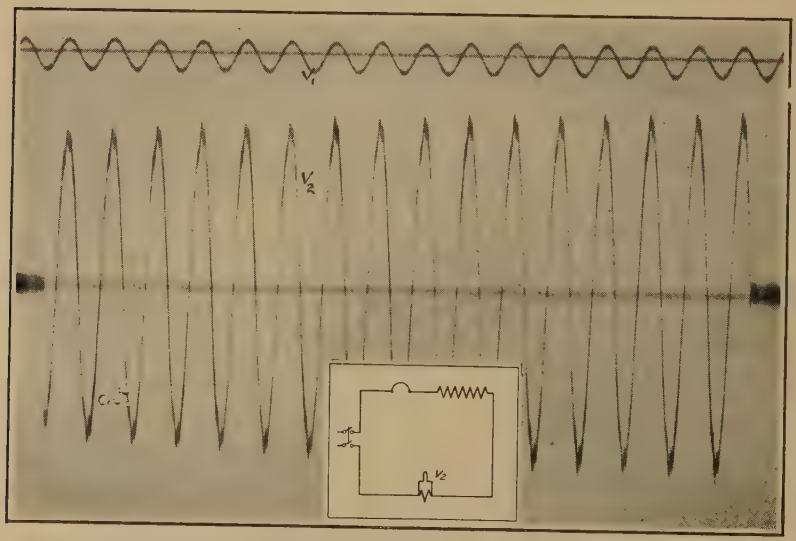


FIG. 205.—A.C. circuit breaker.
E = 130 volts; I = 14.6 amps.; v_1 = 60 ∼.

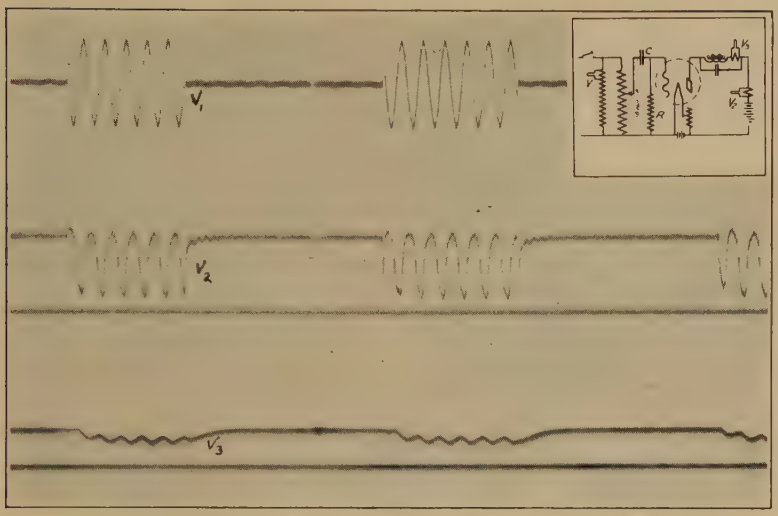


FIG. 206.—Vacuum tube transients.

$v_1 = E_g = 18$ volts, 60 ~. $v_2 = I_p = 6$ mil. amps.; $v_3 = I_f = 0.25$ amps.; $c = 0.09$ mf. Tube: (UV 201-A). Grid leakage = ¼ mega ohm.

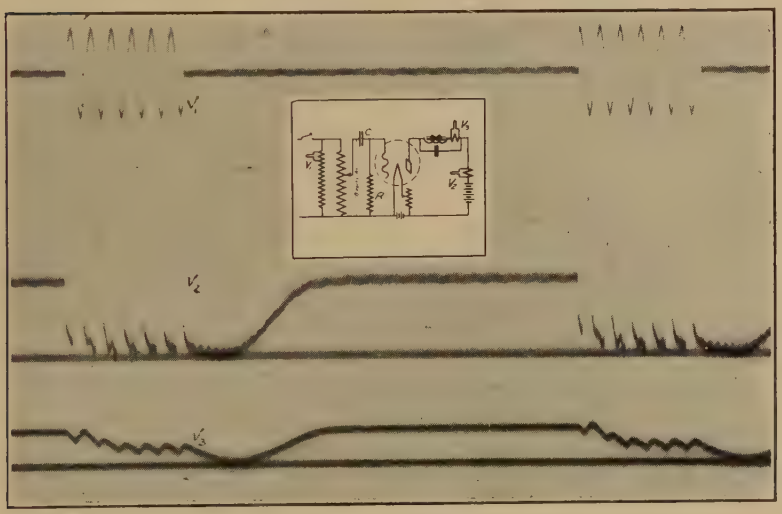


FIG. 207.—Vacuum tube transients.

$v_1 = E_g = 4$ volts, 60 ~. $v_2 = I_p = 7$ mil. amps.; $v_3 = I_f = 0.21$ amps.; $E_p = 80$ volts. Tube: (UV 201-A). No grid leakage condenser. Grid bias = + 4.5 volts.

FIG. 208.—Cathode-ray oscillogram of the hysteresis loop of laminated carbon steel. (For circuit diagram see Figs. 27 and 28.)

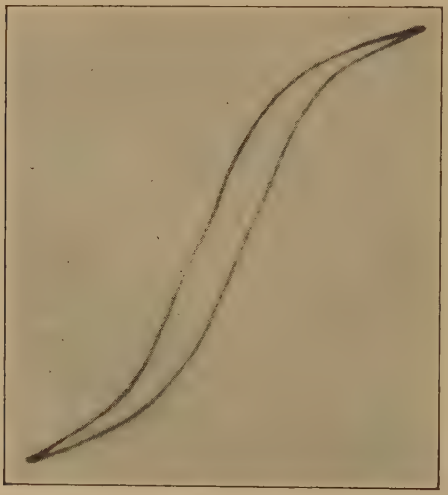
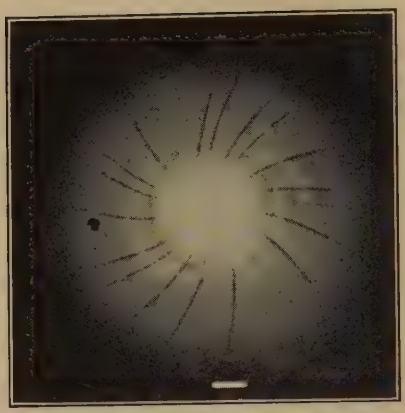
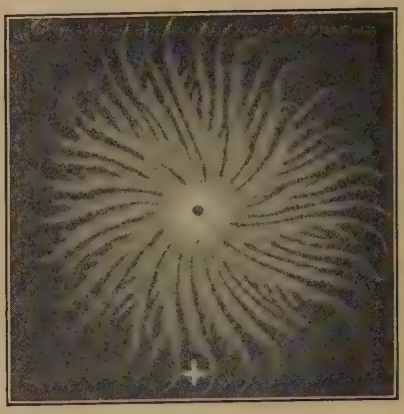


FIG. 209.—Cathode-ray oscillogram of the hysteresis loop of laminated tungsten steel. (For circuit diagrams see Figs. 27 and 28.)

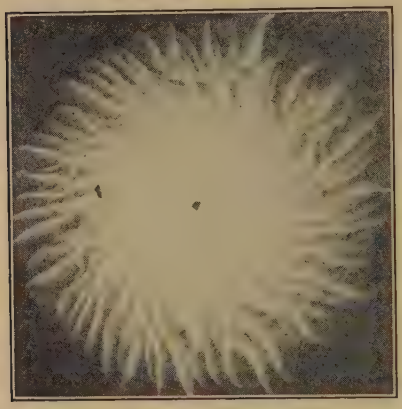
FIG. 210.—Lichtenberg figures or klydonograms. 76 cm. air pressure. 4 cm. spark gap.



Pressure 1.20 cm.; Gap 1.2 cm.; Field N, 12,400 cm.²



Pressure 1.20 cm.; Gap 1.2 cm.; Field S, 12,400 cm².



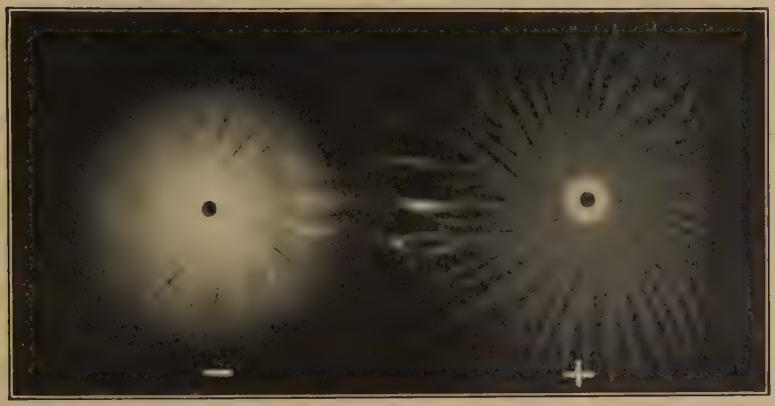
Pressure 0.6 cm.; Gap 0.4 cm.; Field S, 12,200 cm².

Pressure 0.6 cm.; Gap 0.4 cm.; Field N, 12,200 cm².

Pressure 0.7 cm.; Gap 0.5 cm.; Field N, 12,600 cm².

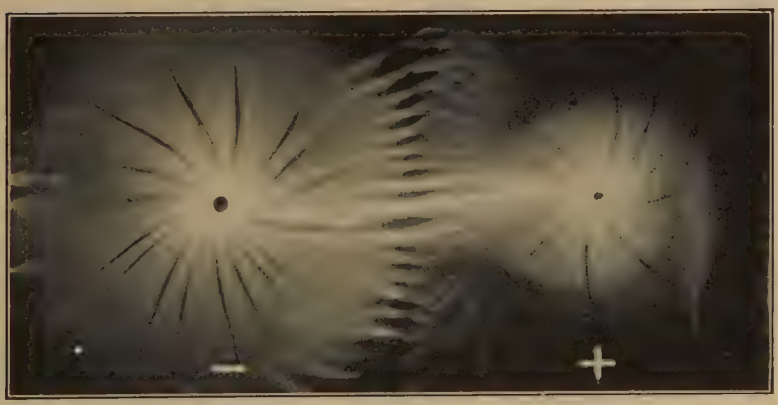
Pressure 0.3 cm.; Gap 0.5 cm.; Field S, 12,500 cm².

FIG. 211.—Lichtenberg figures at reduced air pressures and taken in a magnetic field. Single electrode on photographic plate. Direction of magnetic flux at right angles to photographic surface; S toward reader and N away from reader through printed page.



(*a*) Pressure 15.0 cm.; Gap 1.0 cm.; Field N, 12,500 cm².

(*b*) Pressure 15.0 cm.; Gap 1.1 cm.; Field N, 12,500 cm².



(*c*) Pressure 15.0 cm.; Gap 4.0 cm.; Field S, 12,500 cm².

FIG. 212.—Lichtenberg figures having both electrodes on the same side of photographic plate. Taken under stress of a magnetic field.

Problems and Experiments

1. Take oscillograms of a number of transients in circuits of the types shown in this chapter. In each case obtain quantitative data and prepare a report giving an explanation of the transients appearing in the oscillogram based on the fundamental principles of transient electric phenomena.

2. Find several electric transients in the laboratory under different circuit conditions from those described in the book. For each case draw diagrams of the proposed circuit connections showing the location of the vibrators; make preliminary calculations as to the amount of resistance required in each vibrator circuit; the most desirable speed of the film drum, etc., to give a well-proportioned oscillogram; take the oscillogram; record the quantitative data; develop the film and make prints. Compare the predicted forms of the curves with the photographic record and check the preliminary calculations with the final circuit data. Prepare a report on the transients recorded on the oscillogram.

CHAPTER X

LABORATORY INSTRUCTIONS

To produce good photographic records of transient electric phenomena requires ingenuity, good judgment and careful planning combined with patience and manipulative skill. In principle, oscillographs are simple instruments, but the taking of well-proportioned and clearly defined oscillograms is not an easy task. The very short duration of the phenomena to be recorded and the precise time relation that must obtain between the several operations involved make the production of good oscillograms no mean accomplishment. The optical system, the electric circuits, the mechanical devices and the photographic chemical reactions must all function in a satisfactory manner under highly exacting time-sequence requirements.

Considerable experience in taking simple transients is necessary before problems of more complex nature should be attempted. The most fruitful method for advancing in the art is to study carefully each record directly after it is produced so as to ascertain the cause for any defects and if possible to determine how the quality of the oscillogram might be improved. A close scrutiny of each oscillogram directly after the film or plate has been developed will prove of value not merely for gaining better insight into the cause and effect relations in the several processes involved, but also to reveal unexpected results, if any, that might otherwise be overlooked.

The student should take pride in producing the best possible records. Not only must the lines be distinct and sharply outlined, but the curves must be of good proportion both in amplitude and wave length as well as in position on the film. The more difficult oscillograms, and

sometimes even simple transients, require much study and planning in preparation for the taking of the oscillogram. In some cases preliminary experimental tests are made to gain information on the quantitative values of the factors involved. The operator must predetermine most of the quantitative features and should visualize the desired photographic record while adjusting the oscillograph and accessory apparatus.

Careful work at all stages of the process is an essential requirement. Records that may be all that could be desired when taken from the oscillograph may easily be marred or ruined by careless handling of the films in the dark room or in developing and printing.

Care and Adjustment of the Oscillograph.—While the vibrating mirror type, of oscillographs as described in Chap. II operate on the same basic principle, the design and accessory devices differ greatly and hence only general instructions are given.

The optical train requires great care and constant watchfulness when making the adjustments. If an electric arc is used, painstaking care is required in having the bright spot on the carbon electrode stable before making an exposure. If the electrodes are slightly too far apart, the brilliancy of the hot spot is reduced and the arc may break at any moment. If the electrodes are too close together the arc may sing, producing bright and dull spots on the photographic record. The optical train should be so adjusted that the beam of light reflected by the vibrating mirror comes from the most brilliant spot on the carbon electrodes.

Dust and dirt on the various surfaces through which the beam of light must pass may cause no end of trouble. This applies especially to the windows of the oil-filled cells. Probably the easiest way to clean a small glass surface is by rubbing with a small tuft of absorbent cotton spun on the sharpened end of a match. Sometimes the soft wood of a match sharpened to a long, thin wedge may be used as a wiper to give the desired polish.

When the light strikes the mirrors at an angle, and also if the mirror glass is not uniform, the spot produced shows spectral coloring. The best remedy is to keep all light angles at minimum values, and where coloring appears in the light spot adjust it to show brightest on the blue end of the spectrum. Make adjustments so that the spot producing the photographic record is well in the blue end. The ground glass, which may be placed in the position occupied by the film during exposure, should be used while making final adjustments on the optical train and the amplitude of the vibrator deflections.

The electrical system may be considered as consisting of all circuits directly or indirectly related to the electrical phenomena under observation. For quantitative work the elements for the current vibrators and the multiplying resistances for the voltage vibrator must be calibrated. For qualitative experiments and especially for student practice work, the potentiometer circuits shown in Fig. 213 are more convenient and enable the operator to start at zero amplitudes and gradually increase the currents passing through the vibrators and thus avoid the danger of damaging the instrument by excessive currents. All vibrators should be properly fused except when very low voltages are measured, in which case the fuse resistance would produce errors in the results. For low-voltage measurements the fuse may be replaced by a fine wire, but special care must be exercised to guard against excessive currents that would burn out the element.

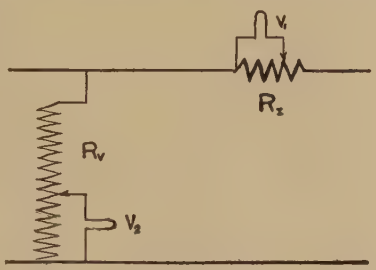


FIG. 213.—Potentiometer circuit diagram for regulating vibrator currents.

The photographic phase of the work begins at the revolving drum carrying the film or bromide paper and includes the several accessory devices employed for opening and closing the shutter at predetermined time instants.

In most cases it is desirable to start the exposure just as the end of the film or paper on the revolving drum passes the slot opening. The transients to be recorded should be so timed that exposure begins when 1 or 2 cm. of film has moved past the slot in the drum case and ends before the whole length of film has been exposed. The inertia of the shutter mechanism must be considered when adjusting the time sequence of opening and closing for different speeds of

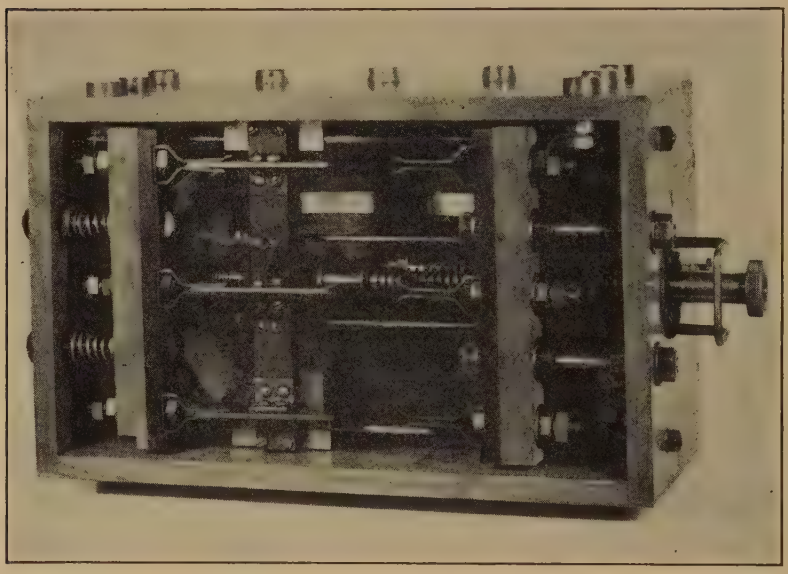


FIG. 214.—An electrically controlled switch for varying the time lag on shutter operations.

the revolving drum carrying the film. The lead required for increase in speed is usually obtained by advancing the setting of the shutter-operating contacts, thus starting the mechanism a fraction of a second before the end of the film passes the slot on the drum case. This angular advance must be proportional to the speed. Little or no change is necessary for speeds below 200 r.p.m., but for higher speeds an advance of about 15° is required for each 100 r.p.m. increase in speed. As shutter mechanisms differ in design

the proper time allowance must be determined for each instrument.

An electrically controlled switch, operated by the shutter contacts, Fig. 214, has proved to be a convenient and reliable device for securing the precise sequence of operations so that the electric transient to be recorded will occur during the period the shutter is open and more particularly will start when a predetermined point on the film is passing the open slot. A variable inductance in the switch-control circuit enables the operator to secure the desired time lag between the opening of the shutter and the operation of the switch. If the film speed is changed, a corresponding variation must be made in the inductance; the lower the speed the greater the required inductance.

The following suggestions for overcoming some of the difficulties that frequently confront students when gaining experience in operating oscillographs may be helpful:

1. Sluggish or erratic action of the vibrators may be due to a number of factors:

a. Insufficient tension on the element.

b. Element touching the frame of pole pieces.

c. Dirt or magnetic material between element and frame.

d. Mirror attached to only one side of element.

e. Evaporation of damping fluid until it is too thick or too low in the cell.

f. Frame of element not fastened securely to cell.

g. Poor connections in circuit, as poor contacts in switches, binding posts and sliding rheostats. The vibrator element can readily be tested by a small magneto set. The magneto current will cause the vibrator to oscillate if the field is excited and will not damage the fuses or the element.

h. Small vibrations occurring on all vibrators and records may be due to vibration of the drum. Peculiar vibrations of one vibrator may be caused by the element touching the frame.

2. Burning out of fuses is in most cases due to carelessness in making adjustments. Start with as small a current in the vibrator element as possible and increase slowly until the desired amplitude of oscillation is obtained. Sometimes the fuse does not make good contact with the fuse holder, producing a high resistance or an open circuit.

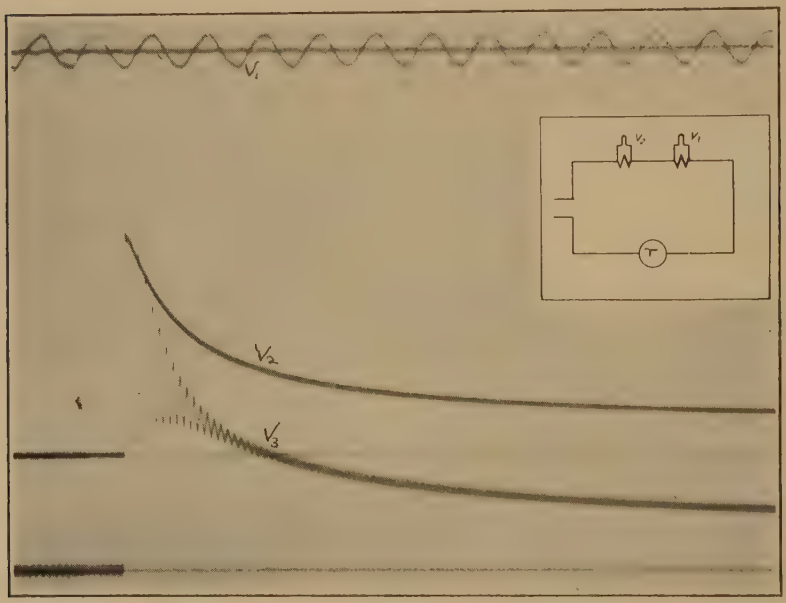


FIG. 215.—Oscillations of undamped vibrator of the three-element oscillograph shown in Fig. 17.
$E = 102$ volts; $I = 1.5$ amp.; $T = 2{,}000$-watt tungsten lamp v_2, properly adjusted by damping; v_3, undamped vibrator; $v_1 = 100\sim$.

3. Difficulty in producing a zero line that coincides with the zero position of the transient may be due to several factors:

a. A sluggish vibrator, slowly returning to its final position.

b. Poor adjustment of the prism so that the narrowing of the light beam for zero-line exposure occurs only on one side of the light spot.

c. Small magnetic particles in the damping oil and on or near the element may change in position when the field

is demagnetized. The remedy is to leave the field excitation unchanged until the zero-line exposure has been completed.

4. The nature of the damping fluid in the vibrator element is of considerable importance. The vibrator must be damped so as to eliminate the mechanical vibrations of the element itself that otherwise would follow any sudden change in the position of the mirror, as illustrated in Fig. 215. The liquid used for damping purposes must be clear and colorless and of sufficient viscosity and should not deteriorate with age. A mixture of castor oil and turpentine has been widely used, but as the turpentine evaporates quite readily the thickness and viscosity as well as the transparence of the liquid change, making frequent renewals necessary.

Nujol serves very well as a damping fluid and for this purpose is in many respects superior to the castor oil-turpentine mixture.

Transients Photography.—The oscillograph records on a photographic film or bromide paper the current-time and voltage-time relations of transient electric phenomena. Special sized films made for this purpose are on the market, but some of the latter oscillographs are designed for using ordinary roll films of standard sizes.

Except for high-speed work the Eastman P.M.C. No. 1 regular bromide paper can be used directly on the oscillograph in place of the films. This reduces the expense and saves time in the dark room. The use of bromide paper in place of films is recommended for student work, especially when making trial exposures. If an excellent bromide paper record is obtained, it can be photographed and duplicated by printing. Most of the oscillograms illustrating the text of this book were taken on bromide paper.

In photographing electric transients the time of exposure is in most cases necessarily of very short duration and the spot of light moves at high speed over the film surface. As a consequence the lines are frequently very faint and

it becomes necessary to develop the film to the utmost in order to gain the greatest possible contrast. Hence special care must be taken to guard against fogging. Unless the dark-room red light is unusually safe, it is best to work in complete darkness until the film has been in the developer for at least a minute.

Films made especially for the oscillograph are packed with a sheet of black paper next to the emulsion side. The paper protects the film against finger marks or bruises that would mar or ruin the oscillogram. During the loading process the film and its protecting paper should be handled as a unit. Not until the film is securely fastened to the drum should the black paper be removed. The film should be wound around the drum in the opposite direction to the rotation of the drum when an exposure is made, so as to eliminate the possibility of having the film end catch on the inner side of the drum box. The emulsion side of the film must not be touched or allowed to come in contact with any object until the exposure has been made and the developing process, including the drying stage, is completed. The same care should be exercised in removing the film from the drum after exposure, as in loading.

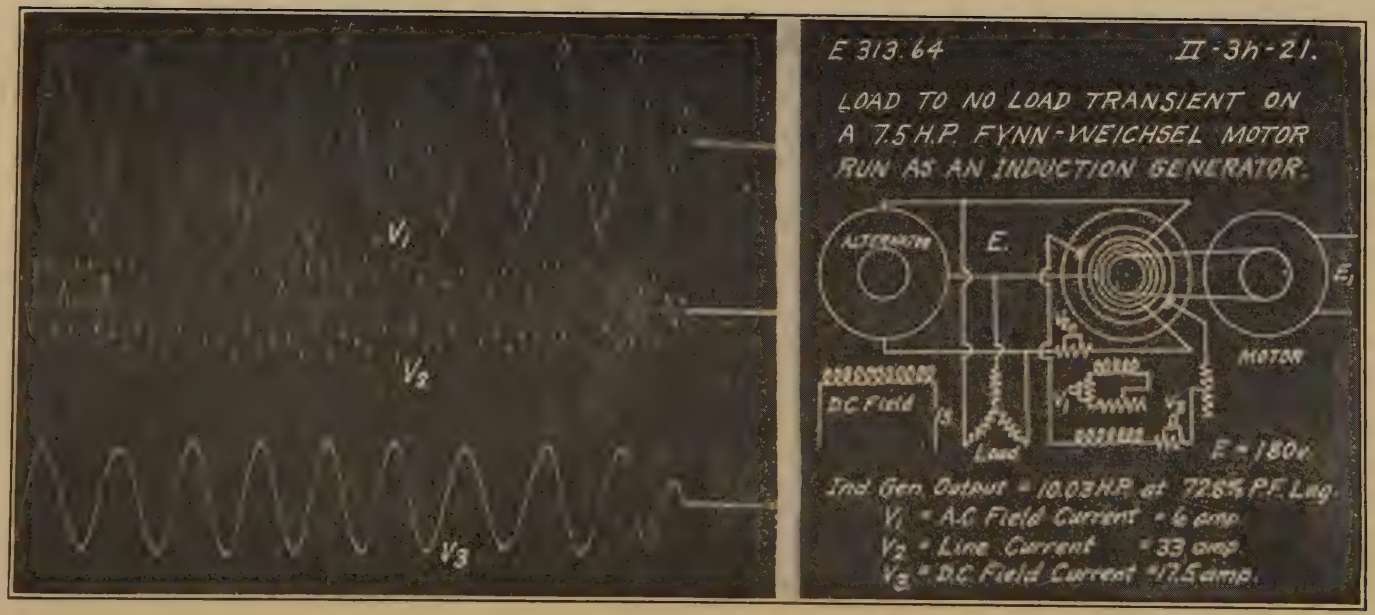
The film should be developed so as to obtain the sharpest contrast possible. It is best to wet the film by passing it through a tray of fresh water a few times before dipping it into the developer tray. Pass the film through the developer several times before leaving it wholly immersed and continue the process by rocking the tray at intervals. The film should be developed to the fogging point. A good rule to follow is to develop until the image can easily be seen from the back side of the film. The emulsion side will then have a gray appearance but the gray tone will disappear in the fixing bath. When the film is fully developed it should be rinsed in fresh water and then placed in the fixing bath, making sure that the entire film is covered by the solution. The film should remain in the fixing bath at least 5 min. after the last visible trace of unreduced

silver salts has disappeared and 30 to 45 min. in the hypo solution is not excessive.

After fixing, the film should be thoroughly washed in clear running water for at least 20 min. Before the film is removed from the rinsing water both sides should be rubbed gently with a large tuft of absorbent cotton. If this cleaning is not well done the films may show white streaks due to sediment deposited on the film in the fixing bath. After the film has once been dried it is very difficult to remove the streaks.

The film should be hung to dry in a place free from dust and at ordinary room temperatures. A gentle breeze of slightly warmed air will accelerate the drying. Too much heat will swell the wet emulsion and cause wrinkles. The film should remain in the same position until thoroughly dry, as any change may cause drops of water to run over partly dried portions, thereby causing streaks and wrinkles. After the drying is completed the film should be labeled with india ink near one end and placed in file or finished for printing. All quantitative data relating to the oscillogram, including a circuit diagram showing the exact location of the vibrator, a title and a filing number, should be placed on or attached to each film. In order to secure uniformity and to avoid blurring the oscillographic record a tab of tracing cloth carrying the data and circuit diagram, title and filing number may be attached to each film, as illustrated in Fig. 216. The tab should be cemented to the dull side of the film by a good glue and thus made a permanent part of the record. All figures and letters on the tab when attached to the film should read correctly from left to right when looked at from the dull side of the film.

As the sharpest possible contrast is desirable in oscillograms, a glossy, strong contrast paper will be most desirable for prints. Azo No. 4, glossy, gives good results. The paper may be bought in rolls and cut to conform with the length of the film including the tab. A printing box of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II-3h-21.
LOAD TO NO LOAD TRANSIENT ON
A 7.5 H.P. FYNN-WEICHSEL MOTOR
RUN AS AN INDUCTION GENERATOR.
ALTERNATOR
E.
MOTOR
D.C. Field
Load
F = 180~
Ind. Gen. Output = 10.03 H.P. at 72.6% P.F. Lag.
V1 = A.C. Field Current = 6 amp.
V2 = Line Current = 33 amp.
V3 = D.C. Field Current = 17.5 amp.

FIG. 216.—Data tab attached to oscillogram.

proper dimensions for the finished film, with a mask for a white border on the print, will be found useful.

Ability to determine the time of exposure must be gained largely by experience, since it varies with the film density, the quality and intensity of the light used and the kind of printing paper. To avoid waste, a small piece of paper may be exposed to a portion of the film showing the record and developed as a trial. The proper exposure should require about 20 sec. or longer to reach the desired density. For films lacking in contrast, that is, having very faint lines, the best results are obtained by slightly underexposing followed by overdeveloping the print.

When the proper density is obtained the print should be quickly rinsed in fresh water and then immersed in the hypo bath, where it should remain 15 min. or longer. After fixing it should be thoroughly washed in running water for at least 20 min.

Best results are obtained by drying the prints on ferrotype plated. The plates must be thoroughly cleaned by sprinkling with benzine in which a small amount of paraffin has been dissolved (about 5 c.c. of paraffin to a liter of benzine), then rubbing with a large tuft of absorbent cotton until the surface is dry and shows a high polish. Place the wet prints face down on the polished surface of the ferrotype plates, cover with a blotter and then squeeze the prints in perfect contact with the plates by the roller. If properly done, the dry prints may be peeled from the plates with ease. If the prints show a tendency to stick, this is probably due to poor hypo, too little acid in the fixing bath or too high temperature when drying.

Any standard developer giving good contrast may be used. Pyro developer gives good results with films but deteriorates rapidly, stains the fingers and can not be used for papers. Eastman Formula D 11, an elon-hydrochinon developer, gives good results for both films and papers, produces good contrast and has excellent keeping qualities. Prepared developers, like Eastman's Special developer

tubes, may be used with good results. A little potassium bromide solution added to the developer will help to retard fogging and allow more contrast to be gained in most films.

Films of value should be kept in a convenient file. An envelope for each film together with prints will afford protection and keep them together. In order that any film may be readily located, some filing system must be used. Unless a simple system for local use is desired, the Dewey Decimal system, so extensively used in libraries, can be used to advantage. The title and corresponding filing number should be placed both on the envelope and on the enclosed film.

INDEX

A

Alternator, armature and field transients, 76, 88
  armature reactance, 86
  armature reaction, 86
  synchronous reactance, 86
Angles, 133
  electric line oscillations, 136, 137, 138, 139
Arcing, 177
Artificial electric lines, 131
Attenuation constants, 55

B

Breaking field circuit, 62

C

Capacitance, 9
Capacity, 9
Carbon lamps, starting transients, 168
Compound circuits, 163
  oscillations of, 164
Condensance, 9
Conductance, 9
Conductivity, 9
Corona, 181
Coupled circuits, 194
  dielectrically coupled, 198
  inductively coupled, 196

D

Damping factor, 122, 158
Decrement, 201
Dielectric circuit, 4
  condensance, 6
Dielectric field of circuit, 5
  field of single conductor, 5
  flux, 4, 6
Dielectric energy, 9
  field intensity, 9
  flux density, 9
  gradient, 9
Direct-current motor-field transient, 62
Dissipation constant, 55, 122
Double energy transients, 106
  circuits, 122

E

Elastance, 9
Elastivity, 9
Electric circuit, 7
  energy in a dielectric field, 8
    a moving body, 8
  field of circuit, 7
    conductor, 7
Electric current, 9
  density, 9
  gradient, 9
  power, 9
Electric-line oscillations, 131
Exponential curve, 57, 58
Euler's equation, 111

F

Farad, 9
Frequency, distributed R. L. G. and C., 119, 141
  massed R. L. G. and C, 141
Frequency of oscillation, 108, 112, 120
Fröhlich's formula, 170

G

Galvanometer, six-element, 28
three-element, 25
General transmission-line equations, 145
Gilbert, 9

H

Henry, 4

I

Impulses, 162
Inductance, 9
Inductance voltage, 9
Initial transient values, 59
Intermittent arcs, 177

J

Joule, 9

K

Kirchoff's laws, 48, 110
Klydonograms, 221
Klydonograph, 40, 42

L

Laboratory instructions, 225
adjustment of the oscillograph, 226
transients photography, 231
Length of line, 143
Lichtenberg figures, 43, 222, 223
Lumpy line, 132

M

Magnetic circuit, 2
field of circuit, 3
single conductor, 3
flux, 2, 9
inductance, 3
permeability, 3
Magnetic reluctance, 3
Magnetic energy, 9
field intensity, 9
flux density, 9
gradient, 9
Magnetomotive force, 9

N

Natural admittance, 108
impedance, 108
period of oscillation, 140

O

Ohm's law, 2, 48
Oscillator generator, 32
Oscillograms, 34, 47, 205
cathode-ray oscillogram, 220
Oscillograph, 16
Braun-tube, 29
cathode-ray, 27, 30
Duddell, 16
Dufour, 32
Osiso, 18
six-element, 27
three-element, 19, 23

P

Permeability, 9
Permittance, 9
Permittivity, 9
Phase angles, 133
Polyphase short circuits, 76
Power surges, 145
Power transfer constants, 158
factor, 158

R

Reactance curves, 187, 188
Reluctance, 9
Reluctivity, 9
Resistance, 9
Resistivity, 9
Resonance, 184
cumulative, 202

Resonance, current, 187
growth of decay, 201
in high speed signaling, 202
multiplex, 201
sharpness of, 186
voltage, 184

S

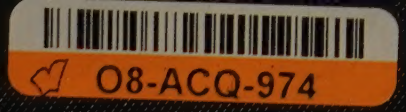
Single-energy short circuits on polyphase alternators, 99
single-phase alternator, 98
Single-energy transients—alternating currents, 66
direct currents, 46
Space angles, 133
Spark gaps, horn gap, 11
impulse ratio, 15
impulse spark-over, 14
needle gap, 10
sphere gap, 11, 12
time lag, 12
Specific electric power, 9
Standing waves, 136
Surge or natural impedance and admittance, 108
Surges or traveling waves, 131
Susceptance curve for parallel circuits, 193

T

Three-phase single-energy transients, 70
Time constant, 53
Traveling waves, 152
Tungsten lamps, starting transients, 168

V

Variable circuit constants, 166
C-circuit constant, 182
G-circuit constant, 176
L-circuit constant, 170
R-circuit constant, 176
Voltage and current oscillations, 145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